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PHYSICAL REVIEW.

ON THE VISCOSITY OF CERTAIN SALT SOLUTIONS.

By B. E. Moore.

THE subject of Viscosity was first taken up by Poiseuille,[1] who used a method depending upon the transpiration of the liquid through capillary tubes. Coulomb, in studying the same subject, observed the damping of a magnetic needle or bar when vibrating in the liquid investigated. This method was also extensively used by O. E. Meyer,[2] Grotian,[3] and others. Still a third method has been developed by Helmholtz, who placed the solution to be studied in a hollow sphere, and observed the behavior of the sphere when oscillated.

These methods have led to very different results. König[4] modified the method of Coulomb and used in the place of the swinging rod, a sphere, the equation of motion of which Kirchhoff had solved, and for which the theory can be completely developed. For calculation König made use of the expression by Kirchhoff as extended and completed by Boltzman, and reached the conclusion that the values obtained for viscosity by Coulomb's method were in complete agreement with the results obtained by allowing the liquid to flow through a capillary tube. Though the agreement

[1] Memoirs des Savants Étrangers, T. IX. Pogg. Ann., Vol. LVIII., p. 434.
[2] Pogg. Ann., Vol. CXIII., pp. 55, 193, 383. [3] Pogg. Ann., Vol. CLVII., p. 130.
[4] Wied. Ann., Vol. XXXII., p. 193.

of results by the different methods is of great interest and importance, yet observers have generally preferred the original method due to Poiseuille, and that because of its simplicity. The method requires also a considerably smaller amount of the solution, and admits of a much easier and more accurate temperature regulation.

Among the earlier investigations on the subject of viscosity may be mentioned those of Graham,[1] whose results suggest the presence of hydrates in solution. He states further that "slow transpiration and low volatility go together." Later investigations by Rellstab[2] dealt with several organic liquids, while numerous experiments were made by Hübner[3] on the salts of the chloride family. Sprung[4] investigated a great many cases of varying concentration and temperature, the latter ranging from 0° to 60° C.

Hagenbach[5] developed the mathematical formula for transpiration. His expression for viscosity (η) reduces, when correction for the velocity of flow is omitted, to the form known as Poiseuille's formula: $\eta = \frac{\pi}{8} \frac{r^4 hst}{lv}$ Herein r denotes the radius of the capillary; h, the height of pressure column; s, specific gravity, and therefore the product of h and s the pressure; v represents the volume that transpires in time, t; and l is used to designate the length of the capillary tube.

Pribam and Handl[6] have made extensive observations on the viscosity of organic solution and stochiometrical relations of the same. The careful work of Gartenmeister[7] in this field should not be omitted. Grotian[8] was the first to make extended comparisons of viscosity and conductivity of salt solutions. Slotte's[9] investigations cover a number of chromates and he shows that the temperature variation in viscosity, η, can be expressed by a formula, $\eta = c(1 + bt)^n$, where c, b, and n are constants of the liquid.

[1] Royal Society Proceedings, XI., p. 381. 1860.
[2] Inaugural Dissertation, Bonn, 1868.
[3] Pogg. Ann. Vol. CL., p. 248.
[4] Pogg. Ann., Vol. CLIX., p. 1.
[5] Pogg. Ann., Vol. CIX., p. 385.
[6] Wien Ber., Vols. 78, 80.
[7] Zeitschrift die Ph. Chem., Vol. VI., p. 524.
[8] Pogg. Ann., Vol. CLVII, p. 130; Vol. CLX., p. 238. Wied. Ann., Vol. VIII., p. 530.
[9] Wied. Ann., Vol. XIV., p. 13.

Since the announcement of the dissociation theory by Arrhenius, the subject of viscosity of solutions has had a much deeper interest, and experiments have been carried on, both trying to establish some stochiometrical relation, and to establish a relation between viscosity and conductivity. With this idea in view, Arrhenius has made many investigations. In his first experiments[1] he shows that the viscosity is a function of x and y, or $H(x, y)$, where x and y express either percentage of substance in solution, or gram-equivalent per liter. We may say $\eta = H(x, y) = A^x B^y$, where A and B are constants of the solution. For a single salt in solution this reduces to $\eta = A^x$. Wagner[2] and Reyher[3] validified this law for a great many solutions. In Gartenmeister's[4] experiments the Arrhenius exponential formula is not so well satisfied. Reyher found a characteristic relation between the friction or viscosity of free acids and those of the sodium salts, according as a strong or weak acid was present. This variation he made to depend upon the unequal dissociation of the strong and weak acids. The dissociation theory has given great confidence to the belief in a relation of viscosity to conductivity. However, G. Wiedemann,[5] previous to this theory, noticed that the friction which the ions undergo varies in the same way as inner friction; *i.e.* viscosity. The mobility of the ions must then be a function of their fluidity. Arrhenius showed that conductivity did not depend upon fluidity alone. This investigator made a strong point when he showed that the introduction of a non-conducting substance into an electrolyte affected both its conductivity and its viscosity in the same way.[6] Other experimenters, by a direct comparison of conductivities and viscosities, have come to the conclusion that while the conductivities of a series of salts increased, the viscosities in general decreased. However, the increasing and decreasing series stand in no definite ratio to each other.

The following experiments have followed much in the same line.

[1] Zeitschrift die Ph. Chem., Vol. I., p. 285.
[2] Zeitschrift die Ph. Chem., Vol. V., p. 31.
[3] Zeitschrift die Ph. Chem., Vol. II., p. 744.
[4] Zeitschrift die Ph. Chem., Vol. VI., p. 524.
[5] Pogg. Ann., Vol. XCIX., p. 177.
[6] Zeitschrift die Ph. Chem., Vol. IX., p. 487.

The viscosities of a series of salts have been determined, and, in so far as was possible, the conductivities of the same compared with their viscosities.

The method employed was the one due to Poiseuille, the apparatus being similar to that used by Arrhenius. A glass vessel *A* (Fig. 1), of about 24 c.cm. capacity, is connected with two tubes, *a* and *b*, above and below respectively. Each tube has a diameter of about 4 mm. A stopcock closes *a* about 4 cm. from *A*. *b* was joined to a capillary tube *d*, some 40 cm. long. The lower end of the capillary dips into the solution to be studied, contained in a glass vessel, *B*, of about 200 c.cm. capacity. *B* is kept water-tight by means of a rubber cork *e*, and is encased in a brass support *h*. Exactly 50 c.cm. of the solution was always brought into the vessel *B*, and the extremity *c* of the capillary brought into the plane of the upper edge of the brass casing *h*. This was done to secure a constant average height of pressure in all cases. But this was later proven to be an unnecessary precaution, as a change in the length of the capillary, amounting to 18 cm., only made a difference of 2.5 seconds in the transpiration of water at 18° C. The liquid is brought into the vessel *A* to some point *a'* by exhausting the air through a rubber tube *f*. The time of flow was taken between two marks on tubes *a* and *b*. As the mean height of the pressure column is constant, it is evident that the pressure of the different liquids subjected to transpiration varies directly as their specific gravities. So that to obtain the transpiration at constant pressure, it was only necessary to multiply the observed time of flow by the specific gravity of the solution. The time of flow of water at 18° C. was taken as standard. The ratio of the corrected time of flow of a solution to that of water gives the relative viscosity in terms of water as unity. Should the absolute viscosity be desired, it is only necessary to multiply this result by the absolute value of water. Relative values only have been calculated, as the object was to make a comparison of solutions. The temperature was regulated by a water-bath, and two ther-

Fig. 1.

mometers enabled one to note the temperature to tenths of a degree. Hagenbach's correction for velocity of transpiration was sufficiently small to neglect in all cases.

The specific gravities of the solutions were determined by means of a calibrated Mohr's balance, which enabled one to take readings to the fourth decimal place. It was part of the original intention to make the solutions from weighed portions of the salts and of fixed molecular (*e.g.* double normal, normal, half normal, etc.) contents, but the discovery of a mistake in the weight of a crucible made it necessary to interpret in many cases the per cent of salt in solution from tables of percentages and specific gravities. Solutions of K_2CO_3, KOH, NaOH, and K_2SO_4 were made from Kohlrausch's tables.[1] Solutions of Na_2CO_3, $KHCO_3$, $NaHCO_3$, $KHSO_4$ Na_2HPO_4, NaH_2PO_4, $K_2C_2O_4$ and $NaHC_4H_4O_6$ were made from carefully weighed quantities of the salts. Solutions of K_3PO_4, K_2HPO_4, KH_2PO_4 were kindly loaned by Herr Forch. All other solutions were made from Landolt and Bernstein's Tabellen (*2te Auflage*). The specific gravities of solutions of Na_2CO_3 check well with Kohlrausch's tables, but not so well with those of Landolt and Bernstein. The specific gravities of Na_2HPO_4 differ also slightly from the latter tables and in solutions of $K_2C_2O_4$ the difference is quite large. However, specific gravities of $K_2C_2O_4$ interpolated from Landolt and Bernstein's Tabellen give a viscosity curve of doubtful character.

The time of flow was noted over considerable range of temperature from which the time transpiration at 18° was graphically interpolated. By repeated observation the error in time is reduced to about 0.3 seconds. In the following table of observations and results, m denotes the gram-molecular contents; s, the specific gravity; T, the time; and η, the calculated viscosities. In the rows containing neither T nor s, the values of m and η have been graphically interpolated.

[1] Kohlrausch: Leitfaden der practical Physik, 7^te^ Auflage.

TABLE I.

Na_2CO_3				$NaHCO_3$			
m	*s*	*T*	*η*	*m*	*s*	*T*	*η*
0.00	0.9987	197.0	1.000	0.00	0.9987	194.5	1.000
0.25	1.0250	220.6	1.120	0.25	1.0139	205.5	1.057
0.5	1.0517	251.0	1.274	0.5	1.0286	218.0	1.121
1.0	1.0980	328.5	1.667	1.0	1.0575	245.0	1.260
2.0	1.1880	616.2	3.128				
K_2CO_3				$KHCO_3$			
0.00	0.9987	197.0	1.000	0.00	0.9987	194.5	1.000
0.25	—	—	1.059	0.25	1.0146	200.5	1.031
0.273	1.0340	210.0	1.066	0.495	1.0298	206.5	1.062
0.4788	1.0577	223.0	1.132	0.5	—	—	1.065
0.5	—	—	1.138	1.0	1.0581	218.0	1.121
0.9456	1.1100	258.0	1.310	1.98	1.1149	250.0	1.285
1.0	—	—	1.341	2.0	—	—	1.290
1.974	1.2183	381.0	1.934				
2.0	—	—	1.950				

TABLE II.

$NaHSO_4$				NaOH			
m	*s*	*T*	*η*	*m*	*s*	*T*	*η*
0.00	0.9987	194.5	1.0000	0.00	0.9987	194.5	1.0000
0.25	1.0186	206.0	1.059	0.25	1.0099	206.0	1.059
0.5	1.0386	214.0	1.100	0.5	1.0212	215.5	1.108
1.0	1.0753	245.0	1.260	1.0	1.0425	240.0	1.234
2.0	1.1475	315.5	1.622	2.0	1.0843	299.0	1.537
4.0	1.2810	559.0	2.874	4.0	1.1551	552.0	2.837
K_2SO_4				8.0	1.2786	1470.0	7.557
0.00	0.9987	194.5	1.000	KOH			
0.1195	1.0165	198.1	1.019	0.00	0.9987	194.5	1.0000
0.125	—	—	—	0.25	—	—	1.025
0.243	1.0328	204.5	1.051	0 456	1.0212	203.0	1.044
0.25	—	—	1.052	0.5	—	—	1.051
0.49	1.0650	214.0	1.100	0.92	1.0433	213.5	1.098
0.50	—	—	1.106	1.00	—	—	1.110
$KHSO_4$				1.82	1.0864	235.5	1.211
0.00	0.9987	194.5	1.0000	2.0	—	—	1.237
0.5	1.0439	209.5	1.075	4.0	1.1793	307.0	1.578
1.0	1.0866	223.5	1.149	6.8	1.2900	452.0	2.324
2.0	1.1712	263.0	1.352				

TABLE III.

Na_2HPO_4			
m	s	T	η
0.00	0.9987	194.5	1.000
0.125	1.0190	211.3	1.086
0.25	1.0366	231.4	1.189
0.5	1.0741	277.5	1.427
NaH_2PO_4			
0.00	0.9987	194.5	1.000
0.25	1.0184	209.3	1.076
0.5005	1.0391	230.0	1.182
1.001	1.0776	274.0	1.409
2.002	1.1677	450.0	2.313
KH_2PO_4			
0.00	0.9987	194.5	1.000
0.25	1.0220	205.5	1.057
0.5	1.0442	223.0	1.146
1.0	1.0885	254.0	1.306
H_3PO_4			
0.00	0.9987	194.5	1.000
0.25	1.0120	207.0	1.064
0.5	1.0251	222.3	1.143
1.0	1.0508	255.0	1.311
2.0	1.1022	338.3	1.739

Na_3PO_4			
m	s	T	η
0.00	0.9987	194.5	1.000
0.125	—	—	1.098
0.14	1.0222	214.9	1.105
0.25	—	—	1.220
0.276	1.0440	242.3	1.246
0.50	—	—	1.504
0.54	1.0860	307.2	1.579
K_3PO_4			
0.00	0.9987	194.5	1.000
0.125	1.0227	208.5	1.070
0.25	1.0471	219.0	1.126
0.5	1.0933	252.5	1.298
1.0	1.1805	342.2	1.759
K_2HPO_4			
0.00	0.9987	194.5	1.000
0.125	1.0167	202.0	1.039
0.25	1.0343	213.0	1.095
0.5	1.0700	234.0	1.206
1.0	1.1383	300.0	1.542
2.0	1.2633	449.0	2.309

TABLE IV.

$Na_2C_4H_4O_6$				$NaKC_4H_4O_6$			
m	*s*	*T*	η	*m*	*s*	*T*	η
0.00	0.9987	194.5	1.000	0.00	0.9987	194.5	1.000
0.141	1.0185	209.0	1.075	0.20	1.0273	212.5	1.092
0.25	—	—	1.148	0.25	—	—	1.112
0.281	1.0368	226.0	1.162	0.40	1.0547	231.0	1.188
0.5	—	—	1.335	0.50	—	—	1.252
0.562	1.0730	269.0	1.383	0.789	1.1087	287.0	1.476
1.0	—	—	1.823	1.0	—	—	1.679
1.121	1.1427	395.0	2.031	1.656	1.2112	484.0	2.488
$H_2C_4H_4O_6$				$K_2C_4H_4O_6$			
0.00	0.9987	194.5	1.000	0.00	0.9987	194.5	1.000
0.233	1.013	207.5	1.067	0.1815	1.0267	205.5	1.057
0.25	—	—	—	0.25	—	—	1.080
0.467	1.0269	221.5	1.139	0.363	1.0525	220.0	1.131
0.5	—	—	1.160	0.5	—	—	1.195
0.833	1.0542	256.0	1.316	0.7345	1.1036	255.0	1.342
1.0	—	—	1.412	1.0	—	—	1.489
1.478	—	330.0	1.696	1.48	1.2072	363.0	1.866
1.666	1.1092	365.0	1.853	1.5	—	—	(1.883)
$C_4H_6O_4$				$K_2C_2O_4$			
0.00	0.9987	194.5	1.000	0.00	0.9987	194.5	1.000
0.242	1.0076	204.2	1.050	0.25	1.0283	204.0	1.049
0.25	—	—	1.052	0.5	1.0571	214.5	1.103
0.483	1.0166	215.0	1.105	1.0	1.1121	239.5	1.232
0.5	—	—	(1.110)	1.5	1.1663	270.2	1.389
$H_2C_2O_4$				$NaHC_4H_4O_6$			
0.00	0.9987	194.5	1.000	0.00	0.9987	194.5	1.000
0.25	—	—	1.045	0.147	1.0121	205.5	1.056
0.326	1.0146	206.0	1.059	0.25	—	—	1.094
0.5	—	—	1.072	0.294	1.0256	217.0	1.116
0.665	1.0300	217.5	1.118	0.441	1.0386	228.5	1.175
0.848	1.0370	224.5	1.154	0.5	—	—	(1.198)
1.0	—	—	(1.199)				

Curves.

The curves (Figs. 2, 3, 4, and 5) correspond to Tables I., II., III., and IV. of data respectively. The curve for H_2SO_4 (Fig. 3) is drawn from data by Grotian and is given in order to show

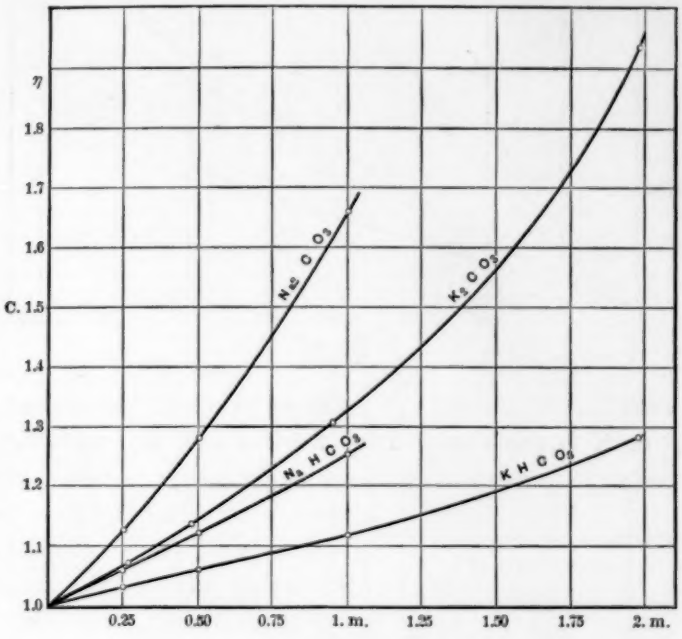

Fig. 2.

the effect of displacing an atom of hydrogen in sulphuric acid. Curves for K_2SO_4 and KH_2PO_4 are not shown, as they nearly coincide with 2 KOH and H_3PO_4 respectively.

Discussion of Results.

The viscosity of sodium salts is invariably greater than that of the potassium salts, and both are greater than that of the corresponding acids. The effect of the hypothetical displacement of the first atom of hydrogen by a given base is generally not so marked as the second displacement by the same base. $NaHC_4H_4O_6$ is an exception. The effect of the second atom of Na and K in the phosphoric acid group (see Fig. 4) is very marked. Whether

the difference is due to the position of the hydrogen atom in the molecule is a question, perhaps, easier to raise than to answer

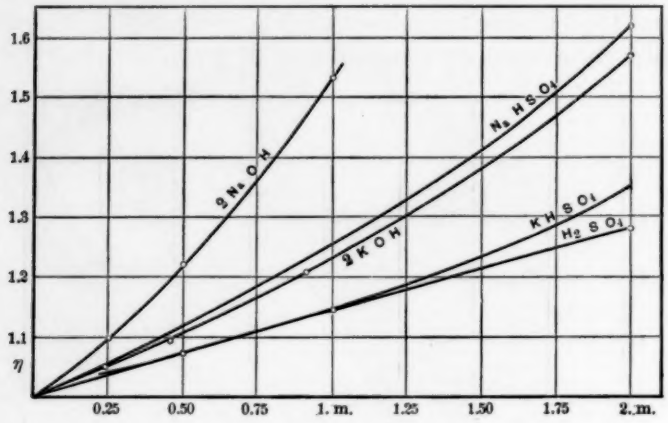

Fig. 3.

satisfactorily. Coincident with the entrance of the second atom of Na or K is the change from marked acid to basic character,

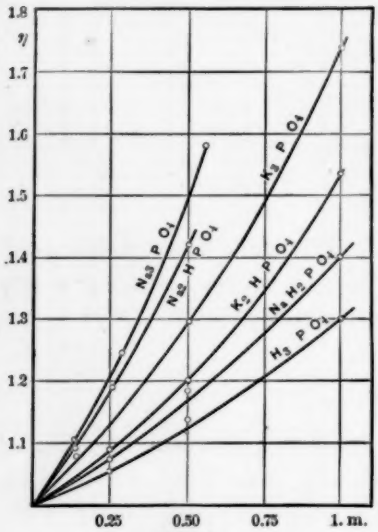

Fig. 4.

which also suggests that the first change in H_3PO_4 ($= H - PO_2 - OH - OH$) took place in the hydroxide radical, the second in the acid radical, and the third again in the hydroxide radical. The curve for KH_2PO_4 lies too near H_3PO_4 to be credited extensively. More confidence is to be placed in the results for H_3PO_4 than KH_2PO_4, as Slotte's observations for H_3PO_4 fall practically upon the curve here given for that acid. Again the neutral phosphate Na_3PO_4 breaks up very easily in the presence of H_2O into the ordinary phosphate Na_2HPO_4 and NaOH, which would make one accept the viscosity curve Na_3PO_4 with some hesitation. So that on the whole it would be rather difficult to draw conclusions

concerning changes in the radicals from the viscosities. So much difficulty does not present itself with the organic compounds. The addition of $(CH_2)_2$ to $C_2H_2O_4$ (=COOH−COOH) (see Fig. 5, curves), giving $COOH-CH_2-CH_2-COOH$, increases the vis-

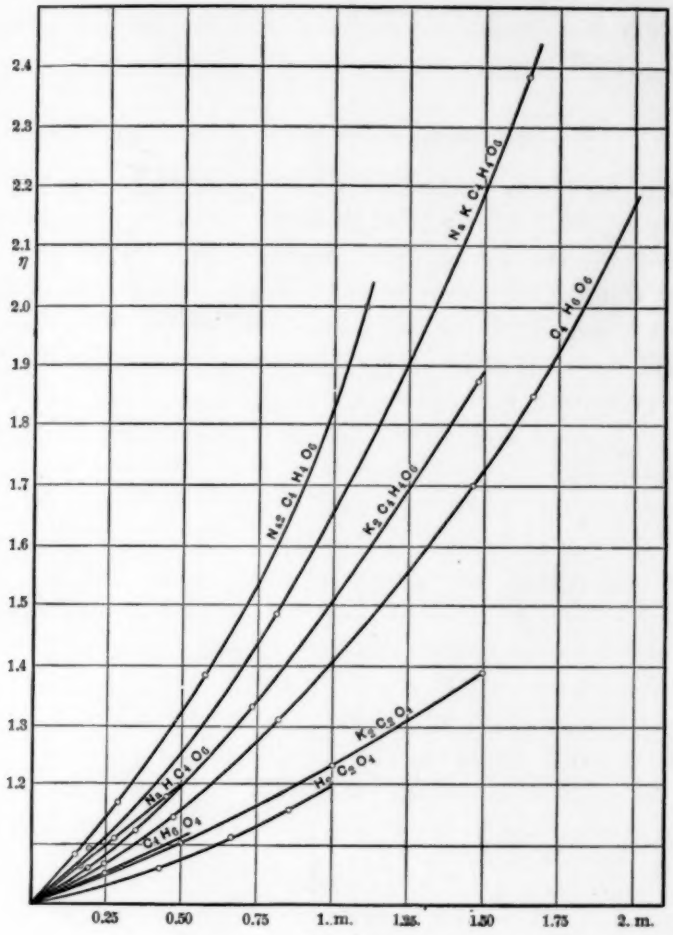

Fig. 5.

cosity over three times as much as the substitution of potassium for hydrogen in $(COOH)_2$. The farther substitution of two hydroxyl radicals for two atoms of H in $(CH_2)_2$ gives also a marked increase, and also greater than the effect of potassium substi-

tuted in $H_2C_2O_4$. Again, a comparison of curves for $H_2C_2O_4$ and $K_2C_2O_4$ with the curves for $C_4H_6O_4$ and $K_2C_4H_4O_6$ shows a marked difference in the effects of potassium on the two salts. In the first pair of solutions potassium entered the carboxyl, giving COOK—COOK, while in the second group the element potassium has worked upon the hydroxide, yielding COOH—CHOK—CHOK—COOH.

Arrhenius Exponential Formula.

When Arrhenius announced the exponential formula, $\eta = A^x$, he only tested it to 1.5 gram-molecule solutions. Wagner, who validified the law for so many solutions, did not go above the normal solution. So that it was thought well to see if such a formula would hold for more concentrated solutions. To test the validity of the law for very dilute solutions, where the law is most serviceable, it would be necessary to limit oneself to very narrow range of and small changes in temperature. It would be imperative to use a bulb *A* (Fig. 1) of smaller volume and a capillary *d* (Fig. 1) of very small bore. The latter invariably clogs and prevents accurate results. Even such precautions would, at the best, only give very small differences, and failure to observe these precautions could not account for the variations from the logarithmic law observed in these experiments. The logarithmic curve, which would represent the viscosities of the more dilute solutions of NaOH, *e.g.* would, if extended to 4 and 8 molecule solutions, give values 36 per cent and 75 per cent too small respectively. No other solutions show so great a divergence. Yet in the double normal solutions the agreement is rarely better than 3 per cent to 5 per cent.

Conductivities and Viscosities.

The values for *A* in the following table have been taken direct from the tables of observations on η, except in the phosphoric acid group, where *A* is reckoned from the equation $\eta = A^x$ and $x = m = \frac{1}{3}$. The conductivities K are those of the normal solutions except when otherwise noted. Only those salts are given

for which the conductivities could be learned. They are divided in four groups corresponding to Tables I., II., III., and IV. of Viscosities respectively.

TABLE V.

Substances.	A	$10^9 \cdot K$	
$\frac{1}{2}Na_2CO_3$	1.274	42.7	
$\frac{1}{2}NaHCO_3$	1.121	37.9	
$\frac{1}{2}K_2CO_3$	1.138	66.2	
$\frac{1}{2}KHCO_3$	1.065	61.3	
$NaOH$	1.234	149.0	
KOH	1.11	171.8	
$\frac{1}{2}Na_2SO_4$	—	47.5	
$\frac{1}{2}NaHSO_4$	1.10	—	
$\frac{1}{2}K_2SO_4$	1.106	67.2	
$\frac{1}{2}KHSO_4$	1.075	173.6	
$\frac{1}{3}Na_3PO_4$	1.305	97.5	
$\frac{1}{3}Na_2HPO_4$	1.260	79.6	$\frac{1}{32}$ normal solution.
$\frac{1}{3}NaH_2PO_4$	1.105	69.8	
$\frac{1}{3}H_3PO_4$	1.08	20.0	
$\frac{1}{2}C_4H_6O_6$	1.160	46.04	
$\frac{1}{2}C_4H_6O_4$	1.110	16.03	$\frac{1}{16}$ normal.
$\frac{1}{2}C_2H_2O_4$	1.070	26.7	
$\frac{1}{2}K_2C_2O_4$	1.100	68.8	

That, while the viscosities in general decrease, the conductivities of a series of salts increase, as noted in the early part of this article, cannot be concluded at all from these salts. This action is noticeable, however, in passing from the sodium salt to the potassium in the first and second group, but when one passes either from sodium carbonate or from potassium carbonate to the acid salts, viscosities and conductivities increase and decrease together. The sulphates behave in the same manner. In the third or phosphate group, in passing from Na_3PO_4 to H_3PO_4, both viscosities and conductivities decrease. In the fourth or organic group, there is an irregularity in the conductivity column. This list, though small, is enough to show that there is little hope for a successful comparison of viscosity and conductivity, without an extended series of observations,

and that, too, with dilute solutions in which the increase in the viscosity of the solvent will be largely due to the ions, as it is the viscosity of the latter alone with which we have to deal in conductivity.

Conclusions.

1. The viscosity of solutions decreases quite rapidly with rise in temperature, but the character of this decrease is very different for different concentrations and for different salt solutions.[1]

2. Stochiometrical relations, though doubtless existing, are neither very definite nor convincing.

3. The Arrhenius exponential formula or law, though affording an excellent method for comparison of viscosities of dilute, even normal, solutions, does not hold good for the more concentrated solutions.

4. More extended observations must be made upon the relation of viscosity to conductivity, perhaps even some new method of comparison arrived at, before the two subjects are placed in their right relation.

[1] Thorpe and Rodgers, Proc. Royal Soc., 55, p. 148, have pointed out that temperatures of equal slopes is an excellent method for comparison of viscosity. These experiments were in a measure completed when the article by Thorpe and Rodgers appeared, and the range of temperature not wide enough to make such a comparison.

NOTES ON THE THEORY OF OSCILLATING CURRENTS.

By Charles Proteus Steinmetz.

§ 1. *Introduction.*

THE object of the following article is to present a short outline sketch of a modification of the method of complex imaginary quantities, applied to oscillating currents. Such oscillating currents have frequently been considered as ordinary alternating currents of very high frequency, and treated as such, while the essential differences between alternating and oscillating currents have been overlooked. An electric current varying periodically between constant maximum and minimum values, that is, in equal time intervals repeating the same values, is called an alternating current if the arithmetic mean value equals zero; and is called a pulsating current if the arithmetic mean value differs from zero. Alternating currents have found very extensive application for light and power. Pulsating currents are the currents given by open coil arc-light machines, or by the superposition of alternating and continuous currents, etc.

Assuming the wave as a sine curve, or replacing it by the equivalent sine wave, the alternating current is characterized by the period or the time of one complete cyclic change, and the amplitude or the maximum value of the current. Period and amplitude are constant in the alternating current.

A very important class are the currents of constant period, but geometrically varying amplitude, that is, currents in which the amplitude of each following wave bears to that of the preceding wave a constant ratio. Such currents consist of a series of waves of constant length, decreasing in amplitude, that is in strength, in constant proportion. They are called oscillating currents in

analogy with mechanical oscillations, for instance of the pendulum, in which the amplitude of the vibration decreases in constant proportion.

Since the amplitude of the oscillating current varies, constantly decreasing, the oscillating current differs from the alternating current in so far that it starts at a definite time, and gradually dies out, reaching zero value theoretically at infinite time, practically in a very short time, short even in comparison with the time of one alternating half-wave. Characteristic constants of the oscillating current are the period T or frequency $N=\frac{1}{T}$, the first amplitude and the ratio of any two successive amplitudes, the latter being called the decrement of the wave. The oscillating current will thus be represented by the product of a periodic

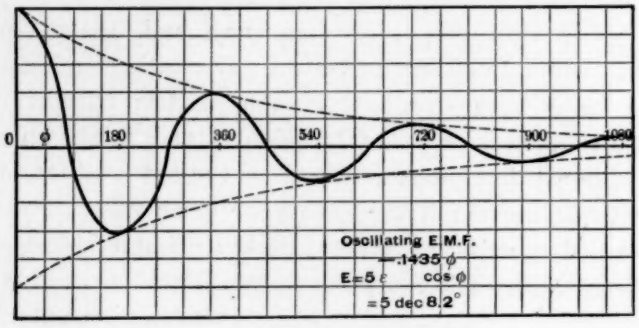

Fig. 1.

function, and a function decreasing in geometric proportion with the time. The latter is the exponential function A^{t-gt}.

Thus, the general expression of the oscillating current is

$$C=A^{t-gt}\cos(2\pi Nt-\hat{\omega}),$$

since $$A^{t-gt}=A^{t}A^{-gt}=c\epsilon^{-bt}.$$

Where ϵ=basis of natural logarithms, the current may be expressed

$$C=c\epsilon^{-bt}\cos(2\pi Nt-\hat{\omega})=c\epsilon^{-a\phi}\cos(\phi-\hat{\omega}),$$

where $\phi=2\pi Nt$; that is, the period is represented by a complete revolution.

In the same way, an oscillating electromotive force will be represented by

$$E = ce^{-a\phi}\cos(\phi - \hat{\omega}).$$

Such an oscillating electromotive force for the values

$$c = 5, \quad a = .1435 \text{ or } \epsilon^{-2\pi a} = .4, \quad \hat{\omega} = 0,$$

is represented in rectangular co-ordinates in Fig. 1, and in polar co-ordinates in Fig. 2. As seen from Fig. 1, the oscillating wave in rectangular co-ordinates osculates the two exponential curves

$$y = \pm ce^{-a\phi}.$$

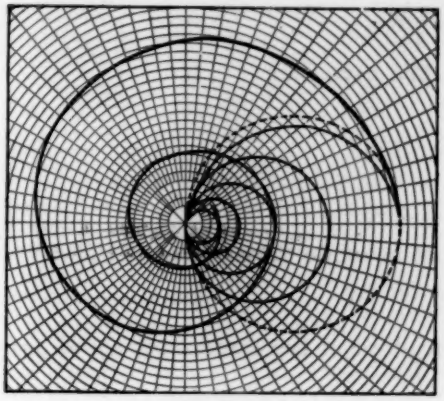

Fig. 2.

In polar co-ordinates, the oscillating wave is represented in Fig. 2 by a spiral curve passing the zero point twice per period, and osculating the exponential spiral

$$y = \pm ce^{-a\phi}.$$

The latter is called the envelope of the oscillating wave, and is shown separately, with the same constants as Figs. 1 and 2, in

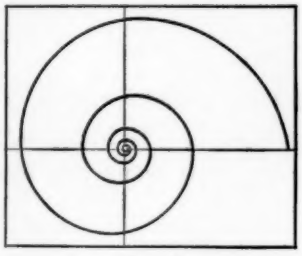

Fig. 3.

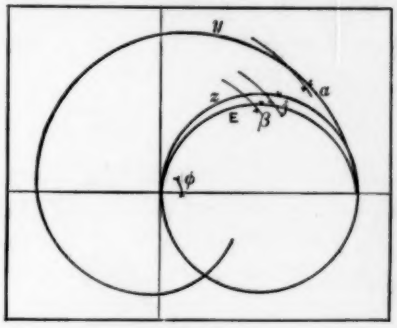

Fig. 4.

Fig. 3. Its characteristic feature is: The angle, which any concentric circle makes with the curve $y = ce^{-a\phi}$, is

$$\tan\alpha = \frac{dy}{yd\phi} = -a,$$

which is, therefore, constant, or in other words: "The envelope of the oscillating current is the loxodromic spiral, which is characterized by a constant angle of intersection with all concentric circles, or all radii vectores." The oscillating current wave is the product of the sine wave and the loxodromic spiral.

In Fig. 4 let $y=ee^{-a\phi}$ represent the loxodromic spiral;

let $z=e\cos(\phi-a)$ represent the sine wave;

and let $E=ee^{-a\phi}\cos(\phi-\omega)$ represent the oscillating wave.

We have then
$$\tan\beta=\frac{dE}{Ed\phi}$$
$$=\frac{-\sin(\phi-\hat{\omega})-a\cos(\phi-\hat{\omega})}{\cos(\phi-\hat{\omega})}$$
$$=-\{\tan(\phi-\hat{\omega})+a\};$$

that is, while the slope of the sine wave, $z=e\cos(\phi-\hat{\omega})$, is represented by
$$\tan\gamma=-\tan(\phi-\hat{\omega}),$$

the slope of the loxodromic spiral $y=ee^{-a\phi}$ is
$$\tan\alpha=-a=\text{constant}.$$

That of the oscillating wave $E=ee^{-a\phi}\cos(\phi-\hat{\omega})$ is
$$\tan\beta=-\{\tan(\phi-\hat{\omega})+a\}$$

Hence, it is increased over that of the alternating sine wave by the constant a. The ratio of the amplitudes of two consequent periods is
$$A=\frac{E_{2\pi}}{E_0}=e^{-2\pi a}.$$

A is called the numerical decrement of the oscillating wave, a the exponential decrement of the oscillating wave, α the angular decrement of the oscillating wave. The oscillating wave can be represented by the equation
$$E=ee^{-\phi\tan\alpha}\cos(\phi-\hat{\omega}).$$

In the instance represented by Figs. 1 and 2, we have, $A=.4$, $a=.1435$, $\alpha=8.2°$.

§ 2. *Impedance and Admittance.*

In complex imaginary quantities, the alternating wave

$$z = e \cos(\phi - \hat{\omega})$$

is represented by the symbol

$$F = e(\cos\hat{\omega} + j\sin\hat{\omega}) = e_1 + je_2.$$

By an extension of the meaning of this symbolic expression, the oscillating wave $E = e\epsilon^{-a\phi}\cos(\phi - \hat{\omega})$ can be expressed by the symbol

$$E = e(\cos\hat{\omega} + j\sin\hat{\omega})\operatorname{dec}\alpha = (e_1 + je_2)\operatorname{dec}\alpha,$$

where $a = \tan\alpha$ is the exponential decrement, α the angular decrement, $\epsilon^{-2\pi a}$ the numerical decrement.

Inductance.

Let r = resistance, L = inductance, and $s = 2\pi NL$ = reactance.

In a circuit excited by the oscillating current,

$$C = c\epsilon^{-a\phi}\cos(\phi - \omega) = c(\cos\hat{\omega} + j\sin\hat{\omega})\operatorname{dec}\alpha = (c_1 + jc_2)\operatorname{dec}\alpha,$$

where $c_1 = c\cos\hat{\omega}$, $c_2 = c\sin\hat{\omega}$, $a = \tan\alpha$.

We have then,

The electromotive force consumed by the resistance r of the circuit

$$E_r = rC\operatorname{dec}\alpha.$$

The electromotive force consumed by the inductance L of the circuit,

$$E_s = L\frac{dC}{dt} = 2\pi NL\frac{dC}{d\phi} = s\frac{dC}{d\phi}$$

Hence

$$E_s = -sc\epsilon^{-a\phi}\{\sin(\phi - \hat{\omega}) + a\cos(\phi - \hat{\omega})\}$$

$$= -\frac{sc\epsilon^{-a\phi}}{\cos\alpha}\sin(\phi - \hat{\omega} + \alpha).$$

Thus, in symbolic expression,

$$E_s = -\frac{sc}{\cos\alpha}\{-\sin(\hat{\omega} - \alpha) + j\cos(\hat{\omega} - \alpha)\}\operatorname{dec}\alpha$$

$$= -sc(a + j)(\cos\hat{\omega} + j\sin\hat{\omega})\operatorname{dec}\alpha;$$

that is,

$$E_s = -sC(a + j)\operatorname{dec}\alpha.$$

Hence the apparent reactance of the oscillating current circuit is, in symbolic expression,

$$S=s(a+j)\operatorname{dec}\alpha.$$

Hence it contains an energy component as, and the impedance is

$$U=(r-S)\operatorname{dec}\alpha=\{r-s(a+j)\}\operatorname{dec}\alpha=(r-as-js)\operatorname{dec}\alpha.$$

Capacity.

Let r=resistance, K=capacity, and $k=\frac{1}{2\pi K}$=capacity reactance. In a circuit excited by the oscillating current C, the electromotive force consumed by the capacity K is

$$E_k=\frac{1}{K}\int Cdt=\frac{1}{2\pi NK}\int Cd\phi=k\int Cd\phi;$$

or, by substitution,

$$\begin{aligned}E_k&=k\int ce^{-a\phi}\cos(\phi-\hat{\omega})d\phi\\&=\frac{k}{1+a^2}ce^{-a\phi}\{\sin(\phi-\hat{\omega})-a\cos(\phi-\hat{\omega})\}\\&=\frac{kce^{-a\phi}}{(1+a^2)\cos\alpha}\sin(\phi-\hat{\omega}-\alpha);\end{aligned}$$

hence, in symbolic expression,

$$\begin{aligned}E_k&=\frac{kc}{(1+a^2)\cos\alpha}\Big\{-\sin(\hat{\omega}+\alpha)+j\cos(\hat{\omega}+\alpha)\Big\}\operatorname{dec}\alpha\\&=\frac{kc}{1+a^2}(-a+j)(\cos\hat{\omega}+j\sin\hat{\omega})\operatorname{dec}\alpha;\end{aligned}$$

hence, $$E_k=\frac{k}{1+a^2}(-a+j)C\operatorname{dec}\alpha,$$

that is, the apparent capacity reactance of the oscillating circuit is, in symbolic expression,

$$K=\frac{k}{1+a^2}(-a+j)\operatorname{dec}\alpha.$$

We have then:

In an oscillating current circuit of resistance r, inductive react-

ance s, and capacity reactance k, with an exponential decrement a, the apparent impedance, in symbolic expression, is:

$$U=\left\{r-s(a+j)+\frac{k}{1+a^2}(-a+j)\right\}\operatorname{dec} a,$$

$$=\left\{r-a\left(s+\frac{k}{1+a^2}\right)-j\left(s-\frac{k}{1+a^2}\right)\right\}\operatorname{dec} a,$$

$$=r_a-js_a;$$

and, absolute,

$$u_a=\sqrt{r_a^2+s_a^2}$$

$$=\sqrt{\left[r-a\left(s+\frac{k}{1+a^2}\right)\right]^2+\left[s-\frac{k}{1+a^2}\right]^2}.$$

Admittance.

Let $C=ce^{-a\phi}\cos(\phi-\hat{\omega})=\text{current.}$

Then, from the preceding discussion, the electromotive force consumed by resistance r, inductive reactance s, and capacity reactance k, is

$$E=ce^{-a\phi}\left\{\cos(\phi-\hat{\omega})\left[r-as-\frac{a}{1+a^2}k\right]-\sin(\phi-\hat{\omega})\left[s-\frac{k}{1+a^2}\right]\right\}$$

$$=cu_ae^{-a\phi}\cos(\phi-\hat{\omega}+\delta),$$

where $$\tan\delta=\frac{s-\frac{k}{1+a^2}}{r-as-\frac{a}{1+a^2}k},$$

$$u_a=\sqrt{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2};$$

substituting $\hat{\omega}+\delta$ for $\hat{\omega}$, and $e=cu_a$, we have

$$E=ee^{-a\phi}\cos(\phi-\hat{\omega}),$$

$$C=\frac{e}{u_a}e^{-a\phi}\cos(\phi-\hat{\omega}-\delta)$$

$$=ee^{-a\phi}\left\{\frac{\cos\delta}{u_a}\cos(\phi-\hat{\omega})+\frac{\sin\delta}{u_a}\sin(\phi-\hat{\omega})\right\};$$

hence in complex quantities,

$$E=e(\cos\hat{\omega}+j\sin\hat{\omega})\operatorname{dec}\alpha,$$

$$C=E\left\{\frac{\cos\delta}{u_a}+j\frac{\sin\delta}{u_a}\right\}\operatorname{dec}\alpha;$$

or, substituting,

$$C=E\left\{\frac{r-as-\frac{a}{1+a^2}k}{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2}\right.$$
$$\left.+j\frac{s-\frac{k}{1+a^2}}{\left(s-\frac{k}{1+a_2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2}\right\}\operatorname{dec}\alpha.$$

Thus in complex quantities, for oscillating currents, we have: conductance,

$$\rho=\frac{r-as-\frac{a}{1+a^2}k}{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2};$$

susceptance,

$$\sigma=\frac{s-\frac{k}{1+a^2}}{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2};$$

admittance, in absolute values,

$$\upsilon=\sqrt{\rho^2+\sigma^2}=\frac{1}{\sqrt{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2}};$$

in symbolic expression,

$$\mathfrak{T}=\rho+j\sigma=\frac{\left(r-as-\frac{a}{1+a^2}k\right)+j\left(s-\frac{k}{1+a^2}\right)}{\left(s-\frac{k}{1+a^2}\right)^2+\left(r-as-\frac{a}{1+a^2}k\right)^2}.$$

Since the impedance is

$$U=\left(r-as-\frac{a}{1+a^2}=k\right)-j\left(s-\frac{k}{1+a^2}\right)=r_a-js_a,$$

we have $\mathfrak{T}=\frac{1}{U};\ \upsilon=\frac{1}{u_a};\ \rho=\frac{r_a}{u_a^2};\ \sigma=\frac{s_a}{u_a^2};$

that is, the same relations as in the complex quantities in alternating current circuits, except that in the present case all the constants r_a, s_a, u_a, ρ, σ, v, depend upon the decrement a.

§ 3. *Circuits of Zero Impedance.*

In an oscillating current circuit of decrement a, of resistance r, inductive resistance s, and capacity reactance k, the impedance was represented in symbolic expression by

$$U=r_a-js_a=\left(r-as-\frac{a}{1+a^2}k\right)-j\left(s-\frac{k}{1+a^2}\right),$$

or numerically by

$$u=\sqrt{r_a^2+s_a^2}=\sqrt{\left(r-as-\frac{a}{1+a^2}k\right)^2+\left(s-\frac{k}{1+a^2}\right)^2}.$$

Thus the inductive reactance s, as well as the capacity reactance k, do not represent wattless electromotive forces as in an alternating current circuit, but introduce energy components of negative sign

$$-as-\frac{a}{1+a^2}k;$$

that means,

"In an oscillating current circuit, the counter electromotive force of self-induction is not in quadrature behind the current, but lags less than 90° or a quarter period, and the charging current of a condenser is less than 90°, or a quarter period ahead of the impressed electromotive force."

In consequence of the existence of negative energy components of reactance in an oscillating current circuit, a phenomenon can exist which has no analogy in an alternating current circuit, that is, under certain conditions, the total impedance of the oscillating current circuit can equal zero:

$$U=0.$$

In this case we have

$$r-as-\frac{a}{1+a^2}k=0;\ s-\frac{k}{1+a^2}=0,$$

substituting in this equation

$$s=2\pi NL\,;\quad k=\frac{1}{2\pi NK}\,;$$

and expanding, we have

$$a=\frac{1}{\sqrt{\frac{4L}{r^2K}-1}}\,;$$

$$2\pi N=\frac{r}{2L}\sqrt{\frac{4L}{r^2K}-1}=\frac{r}{2aL}$$

That is,

"If in an oscillating current circuit, the decrement $a=\frac{1}{\sqrt{\frac{4L}{r^2K}-1}}$, and the frequency $N=\frac{r}{4\pi aL}$, the total impedance of the circuit is zero; that is, the oscillating current, when started once, will continue without external energy being impressed upon the circuit."

The physical meaning of this is: "If upon an electric circuit a certain amount of energy is impressed and then the circuit left to itself, the current in the circuit will become oscillating, and the oscillations assume the frequency $N=\frac{r}{4\pi aL}$, and the decrement $a=\frac{1}{\sqrt{\frac{4L}{r^2K}-1}}$

That is, the oscillating currents are the phenomena by which an electric circuit of disturbed equilibrium returns to equilibrium.

This feature shows the origin of the oscillating currents, and the means to produce such currents by disturbing the equilibrium of the electric circuit, for instance by the discharge of a condenser, by make and break of the circuit, by sudden electrostatic charge, as lightning, etc. Obviously, the most important oscillating currents are those flowing in a circuit of zero impedance, representing oscillating discharges of the circuit. Lightning strokes usually belong to this class.

§ 4. *Oscillating Discharges.*

The condition of an oscillating discharge is $U=0$; that is:

$$a=\frac{1}{\sqrt{\frac{4L}{r^2K}-1}}\qquad 2\pi N=\frac{r}{2aL}=\frac{r}{2L}\sqrt{\frac{4L}{r^2K}-1}.$$

If $r=0$, that is, in a circuit without resistance, we have $a=0$, $N=\frac{1}{2\pi\sqrt{LK}}$; that is, the currents are alternating with no decrement, and the frequency is that of resonance.

If $\frac{4L}{r^2K}-1<0$, that is, $r>2\sqrt{\frac{L}{K}}$, a and N become imaginary; that is, the discharge ceases to be oscillatory. An electrical discharge assumes an oscillating nature only, if $r<2\sqrt{\frac{L}{K}}$. In the case $r=2\sqrt{\frac{L}{K}}$ we have $a=\infty$, $N=0$; that is, the current dies out without oscillation.

From the foregoing we have seen that oscillating discharges, — as for instance the phenomena taking place if a condenser charged to a given potential is discharged through a given circuit, or if lightning strikes the line circuit, — is defined by the equation: $U=0 \operatorname{dec} a$.

Since $C=(c_1+jc_2)\operatorname{dec} a$, $E_r=Cr\operatorname{dec} a$,

$$E_s=-sC(a+j)\operatorname{dec} a,\quad E_k=\frac{k}{1+a^2}C(-a+j)\operatorname{dec} a,$$

we have

$$r-as-\frac{a}{1+a^2}k=0,$$

$$-s+\frac{k}{1+a^2}=0;$$

hence, by substitution,

$$E_k=sC(-a+j)\operatorname{dec} a.$$

The two constants, c_1 and c_2, of the discharge, are determined by the initial conditions, that is, the electromotive force and the current at the time $t=0$.

Let a condenser of capacity K be discharged through a circuit of resistance r and inductance L. Let e=electromotive force at the condenser in the moment of closing the circuit, that is, at the time $t=0$ or $\phi=0$. At this moment the current is zero, that is,

$$C=jc_2,\quad c_1=0.$$

Since $E_k=sC(-a+j)\operatorname{dec} a=e$ at $\phi=0$,

we have $sc_2\sqrt{1+a^2}=e$ or $c_2=\frac{e}{s\sqrt{1+a^2}}$

Substituting this, we have,

$$C = j\frac{e}{s\sqrt{1+a^2}}\operatorname{dec} a, \qquad E_r = je\frac{r}{s\sqrt{1+a^2}}\operatorname{dec} a,$$

$$E_s = \frac{e}{\sqrt{1+a^2}}(1-ja)\operatorname{dec} a, \quad E_h = -\frac{e}{\sqrt{1+a^2}}(1+ja)\operatorname{dec} a,$$

the equations of the oscillating discharge of a condenser of initial voltage e.

Since
$$s = 2\pi NL,$$
$$a = \frac{1}{\sqrt{\frac{4L}{r^2K}-1}},$$
$$2\pi N = \frac{r}{2aL},$$

we have
$$s = \frac{r}{2a} = \frac{r}{2}\sqrt{\frac{4L}{r^2K}-1};$$

hence, by substitution,

$$C = je\sqrt{\frac{K}{L}}\operatorname{dec} a, \quad E_r = jer\sqrt{\frac{K}{L}}\operatorname{dec} a,$$

$$E_s = \frac{er}{2}\sqrt{\frac{K}{L}}\left(\sqrt{\frac{4L}{r^2K}-1}-j\right)\operatorname{dec} a,$$

$$E_h = -\frac{er}{2}\sqrt{\frac{K}{L}}\left(\sqrt{\frac{4L}{r^2K}-1}+j\right)\operatorname{dec} a,$$

$$a = \frac{1}{\sqrt{\frac{4L}{r^2K}-1}}, \qquad N = \frac{r\sqrt{\frac{4L}{r^2K}-1}}{4\pi L},$$

the final equations of the oscillating discharge, in symbolic expression.

§ 5. *Numerical Examples.*

To get an estimate of the numerical values of the constants of oscillating discharges, some cases may be investigated.

A. VERY HIGH FREQUENCY.

A short, straight conductor may be terminated by two balls. The balls represent the capacity, while the conductor represents the resistance and inductance. Without entering into the exact calculation, let the resistance of the conductor

$r = .0001$ ohms; the inductance of the conductor $L = .00025$ millihenry; the capacity of the two balls $K = 10^{-6}$ microfarads. We have then

$$a = 10^{-7}; \text{ angle } \alpha = 5.7 \times 10^{-6}; \; N = 32 \times 10^{4};$$

that is, 320,000,000 cycles per second.

The amplitude is reduced to $\frac{1}{100}$ after the time t_0, or angle ϕ_0, where

$$\epsilon^{-a\phi_0} = .01; \; \phi_0 = 2\pi N t_0;$$

hence,

$$\phi_0 = 4.6 \times 10^7; \; t_0 = .023;$$

that is, after .023 second the oscillation has practically died out; that is, decreased to $\frac{1}{100}$ of its amplitude, after making 7,400,000 complete oscillations.

The equations of the oscillating discharge are in this case, at $e = 10{,}000$ volts initial charge,

$$C = 20j \operatorname{dec} \alpha; \; E_r = .002j \operatorname{dec} \alpha.$$
$$E_s = (10{,}000 - .001j) \operatorname{dec} \alpha; \; E_k = -(10{,}000 + .001j) \operatorname{dec} \alpha.$$

As seen, for a moment 20 amperes flow at a potential of 10,000 volts, representing an instantaneous flow of about 100 K.W.

To make the discharge non-oscillatory, the resistance would have to be $r > 2\sqrt{\frac{L}{K}} > 1000$ ohms; that is, more than 10,000,000 times as much as assumed. That is, a wet string, or similar conductor, will not allow electrical oscillation.

B. Underground Circuit of four Miles in Length, of two Lead-covered Cables consisting of Wire No. 00, B. & S.

$r = 3.3$ ohms; $L = 7.5$ microhenrys; $K = 1.2$ microfarads;

hence,

$$a = .021; \; N = 1670; \; t_0\phi = .021;$$

that is, in .021 seconds, or after 35 cycles, the oscillation has died out to $\frac{1}{100}$ of its initial value. At $e = 2000$ volts initial charge, the equations of phenomenon will be

$$C = 25.3j \operatorname{dec} \alpha; \; E_r = 83.5j \operatorname{dec} \alpha;$$
$$E_s = (2000 - 42j) \operatorname{dec} \alpha \; E_k = -(2000 + 42j) \operatorname{dec} \alpha.$$

The phenomenon will become non-oscillatory; that is,

$$N = 0; \; a = \infty; \text{ for } r = 158 \text{ ohms.}$$

that is, in such cables electric oscillations can take place at comparatively moderate frequency, and of considerable duration.

C. Transatlantic Cable.

Assuming approximately, $r = 40{,}000$ ohms, $L = 30$ h., $K = 1300$ microfarads; we then have $r < 300$, the condition under which electrical oscillations can take place.

If the resistance were $\frac{1}{200}$ the value it is in reality, that is, if we had $r = 200$ ohms, we could have $a = 0.88$, $N = 0.6$, and in this case, at $e = 100$ volts initial charge, the equations of the phenomenon would be, $C = 0.658j \operatorname{dec} a$, $E_r = 131.6j \operatorname{dec} a$, $E_s = (75 - 66j) \operatorname{dec} a$, $E_k = -(75 + 66j) \operatorname{dec} a$; that is, in a transatlantic cable electrical oscillations cannot take place, due to its high resistance. If, however, the resistance were low enough to permit electrical oscillations, that is, less than $\frac{1}{112}$ of what it is in reality, the oscillations would take place extremely slowly, each complete oscillation occupying more than one second.

In reality, due to the capacity not being centralized in a condenser, but distributed over the whole circuit, the phenomenon is more complex, and has to be investigated on the lines of circuits containing distributed capacity.

We see, however, from these instances the enormous range of frequencies, at which electrical oscillations take place, from frequencies of hundred millions of cycles per second to a frequency of more than a second per cycle.

§ 6. *Oscillating Current Transformer.*

As an instance of the application of the symbolic method of analyzing the phenomena caused by oscillating currents, the transformation of such currents may be investigated. If an oscillating current is produced in a circuit including the primary of a transformer, oscillating currents will also flow in the secondary of this transformer. In a transformer let the ratio of secondary to primary turns be p. Let the secondary be closed by a circuit of total resistance, $r_1 = r_1' + r_1''$, where $r_1' =$ external, $r_1'' =$ internal, resistance. The total inductance $L_1 = L_1' + L_1''$, where $L_1' =$ external, $L_1'' =$ internal inductance, total capacity, K_1. Then the total admittance of the secondary circuit is

$$\Upsilon_1 = (\rho_1 + j\sigma_1) \operatorname{dec} \alpha = \frac{1}{\left(r_1 - as_1 - \frac{a}{1+a^2} k_1\right) - j\left(s_1 - \frac{k}{1+a^2}\right)},$$

where $s_1 = 2\pi N L_1 =$ inductive reactance; $k_1 = \frac{1}{2\pi N K} =$ capacity reactance. Let $r_0 =$ effective hysteretic resistance, $L_0 =$ inductance; hence, $s_0 = 2\pi N L_0 =$ reactance; hence,

$$\Upsilon_0 = \rho_0 + j\sigma_0 = \frac{1}{(r_0 - as_0) - js_0} = \text{admittance}$$

of the primary exciting circuit of the transformer; that is, the admittance of the primary circuit at open secondary circuit.

As discussed elsewhere, a transformer can be considered as consisting of the secondary circuit supplied by the impressed electromotive force over leads, whose impedance is equal to the sum of primary and secondary transformer impedance, and which are shunted by the exciting circuit, outside of the secondary, but inside of the primary impedance.

Let r= resistance; L= inductance; K= capacity; hence,

$$s=2\pi NL= \text{inductive reactance},$$

$k=\frac{1}{2\pi NK}$= capacity reactance of the total primary circuit, including the primary coil of the transformer. If $E_1'=E_1'\operatorname{dec}\alpha$ denotes the electromotive force induced in the secondary of the transformer by the mutual magnetic flux; that is, by the oscillating magnetism interlinked with the primary and secondary coil we have $C_1=E_1\mathfrak{T}_1\operatorname{dec}\alpha=$ secondary current.

Hence, $C_1'=p$, $C_1\operatorname{dec}\alpha=p$, $E'\mathfrak{T}_1\operatorname{dec}\alpha=$ primary load current, or component of primary current corresponding to secondary current. Also, $C_0=\frac{p'}{p}E_1'\mathfrak{T}_0\operatorname{dec}\alpha=$primary exciting current; hence, the total primary current is

$$C=C_1'+C_0=\frac{E_1'}{p}\{\mathfrak{T}_0+p^2\mathfrak{T}_1\}\operatorname{dec}\alpha.$$

$E'=\frac{E_1'}{p}\operatorname{dec}\alpha=$induced primary electromotive force. Hence the total primary electromotive force is

$$E=(E'+CU)\operatorname{dec}\alpha=\frac{E_1'}{p}\{1+U\mathfrak{T}_0+p^2U\mathfrak{T}_1\}\operatorname{dec}\alpha.$$

In an oscillating discharge the total primary electromotive force $E=0$; that is,

$$1+U\mathfrak{T}_0+p^2U\mathfrak{T}_1=0;$$

or, the substitution

$$1+\frac{\left(r-as-\frac{a}{1+a^2}k\right)-j\left(s-\frac{k}{1+a^2}\right)}{(r_0-as_0)-js_0}+p^2\frac{\left(r-as-\frac{a}{1+a^2}k\right)-j\left(s-\frac{k}{1+a^2}\right)}{\left(r_1-as_1-\frac{a}{1+a^2}k_1\right)-j\left(s_1-\frac{k_1}{1+a^2}\right)}=0.$$

Substituting in this equation, $s=2\pi NK$, $k=\frac{1}{2\pi NK}$, etc., we get a complex imaginary equation with the two constants a and N. Separating this equation in the real and the imaginary parts, we derive two equations, from which the two constants a and N of the discharge are calculated.

If the exciting current of the transformer is negligible; that is, if $\mathfrak{T}_0=0$, the equation becomes essentially simplified:

$$1+p^2\frac{\left(r-as-\frac{a}{1+a^2}k\right)-j\left(s-\frac{k}{1+a^2}\right)}{\left(r_1-as_1-\frac{a}{1+a^2}k_1\right)-j\left(s_1-\frac{k_1}{1+a^2}\right)}=0;$$

that is,

$$\left(r_1-as_1-\frac{a}{1+a^2}k_1\right)+p^2\left(r-as-\frac{a}{1+a^2}k\right)=0;$$

$$\left(s_1-\frac{k_1}{1+a^2}\right)+p^2\left(s-\frac{k}{1+a^2}\right)=0;$$

or, combined:

$$(r_1-2as_1)+p^2(r-2as)=0,$$

$$r_1+p^2r=2a(s_1+p^2s),$$

$$k_1+p^2k=(1+a^2)(s_1+p^2s).$$

Substituting for s_1, s, k_1, k, we have

$$a=\frac{1}{\sqrt{\frac{4(L_1+p^2L)}{(r_1+p^2r)^2(K_1+p^2K)}-1}},$$

$$2\pi N=\frac{r_1+p^2r}{2a(L_1+p^2L)}=\frac{r_1+p^2r}{2(L_1+p^2L)}\sqrt{\frac{4(L_1+p^2L)}{(r_1+p^2r)^2(K_1+p^2K)}-1},$$

$$E=\frac{E_1'}{p}\{1+p^2U\mathfrak{T}_1\}\operatorname{dec}\alpha,$$

$$C=pE_1'\mathfrak{T}_1\operatorname{dec}\alpha,$$

$$C_1=E_1'\mathfrak{T}_1\operatorname{dec}\alpha,$$

the equations of the oscillating current transformer, with E_1' as parameter.

December, 1895.

AN EXPERIMENTAL STUDY OF INDUCTION PHENOMENA IN ALTERNATING CURRENT CIRCUITS.

By F. E. Millis.

I. *Circuits containing Resistance and Self-induction.*

A DISCUSSION of the current curve produced upon closing a circuit containing an harmonic electromotive force, resistance, and self-induction leads to the well-known equation:

$$i=\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t-\tan^{-1}\frac{L\omega}{R}\right)+ce^{-\frac{Rt}{L}}, \qquad (1)$$

in which

i is the value of the current at any instant,
E is the maximum value of the electromotive force,
R is the Ohmic resistance of the circuit,
L is the self-induction of the circuit,
ω is the angular velocity,
t is the time measured from the instant of closing the circuit,
c is a constant of integration,
e is the Napierian base.

Writers have generally neglected the exponential term in this equation, since as t increases, the effect of that term upon the current rapidly diminishes, and usually becomes inappreciable after the small part of one second. Neglecting this term is equivalent to assuming that the current at once reaches its permanent condition. It is with the curves as modified by this term that the present work deals. If an open circuit be suddenly closed, i and t both being zero at the instant of closing, c in the above equation is determined by Bedell and Crehore to be

$$c=-\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t_1-\tan^{-1}\frac{L\omega}{R}\right), \qquad (2)$$

t_1 being time at which the circuit is closed. For $\frac{E}{\sqrt{R^2+L^2\omega^2}}$ they write I, which represents the maximum periodic value of i;

for $\sin\left(\omega t-\tan^{-1}\frac{L\omega}{R}\right)$ they write[1] $\sin\psi$;

and for $\sin\left(\omega t_1-\tan^{-1}\frac{L\omega}{R}\right)$ they write $\sin\psi_1$.

At the instant of closing the circuit $\psi=\psi_1$, whence, by substitution,

$$i=I\sin\psi-I\sin\psi_1=0; \qquad (3)$$

that is, the initial value of the exponential term is equal and opposite to the value which the current ultimately has when in the same phase as that in which the circuit was closed. The greatest value of this induced or exponential current will occur when $\sin\psi=1$, or when the phase angle at closing the circuit is 90°.

The exponential term would alone give a current curve of the form shown in Fig. 1, the values being measured from the hori-

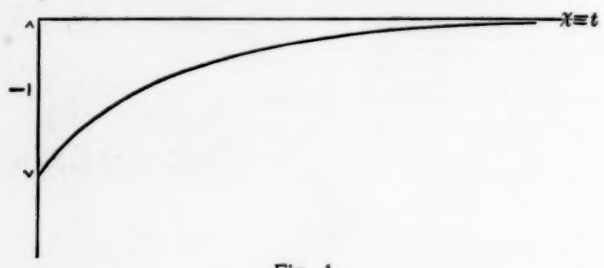

Fig. 1.

zontal axis downward to the curve. This curve compounded with the uniform harmonic current curve represented by the first term of the second member of equation (1), gives a resultant current curve of the general form of c in Fig. 2.

As this short-lived induced current is always to a greater or less extent present whenever the electromotive force in a circuit is suddenly changed or the load varied, it seemed a matter of importance as well as of interest to verify experimentally the above conclusions. This induced current becomes an important factor in

[1] Alternating Currents, Bedell and Crehore, p. 55 (1st ed.).

alternate current working on circuits of large self-induction where the load is subject to great and sudden variation.

The study of small vibrating needles described in the PHYSICAL REVIEW, Vol. III., p. 49, indicated that they may be relied upon to record the current from the instant of closing the circuit.

The experiments here described were made with the needle making 3580 double vibrations per second, mounted as shown in Figs. 1 and 2 of the above article. The coil surrounding the needle had 480 turns, carrying 2.35 amperes; and the magnetic field was produced by an electromagnet with two coils, each having 970 turns and carrying about 10 amperes. The alternating current

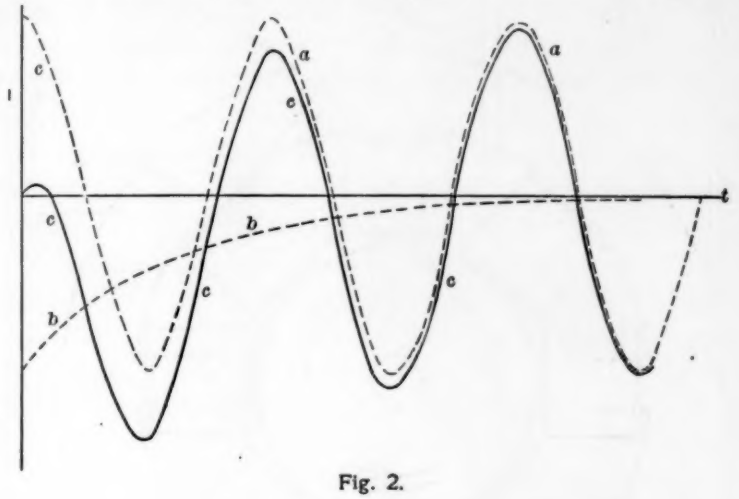

Fig. 2.

experimented upon was produced by a small 50-volt 10-ampere Westinghouse alternator having a smooth armature. Curve II, Fig. 12, in PHYSICAL REVIEW, Vol. III., p. 56, is taken from this alternator when there is no self-induction in the circuit. The resistance of the armature of the alternator was about 0.17 ohms. The entire resistance in the circuit for the curves which follow was 5.1 ohms and the self-induction was 0.0322 henrys.

We have seen that if the current phase is zero when the circuit is closed there will be no induced current, and that the maximum induction is obtained if the circuit is closed when the phase angle is 90°. It was consequently desirable to close the circuit in the

latter phase. To accomplish this, a plank $2\frac{1}{2}$ inches thick was fitted to the top of the alternator, as indicated in Fig. 3. Through this plank holes are bored to admit the electromagnet *SN*, the

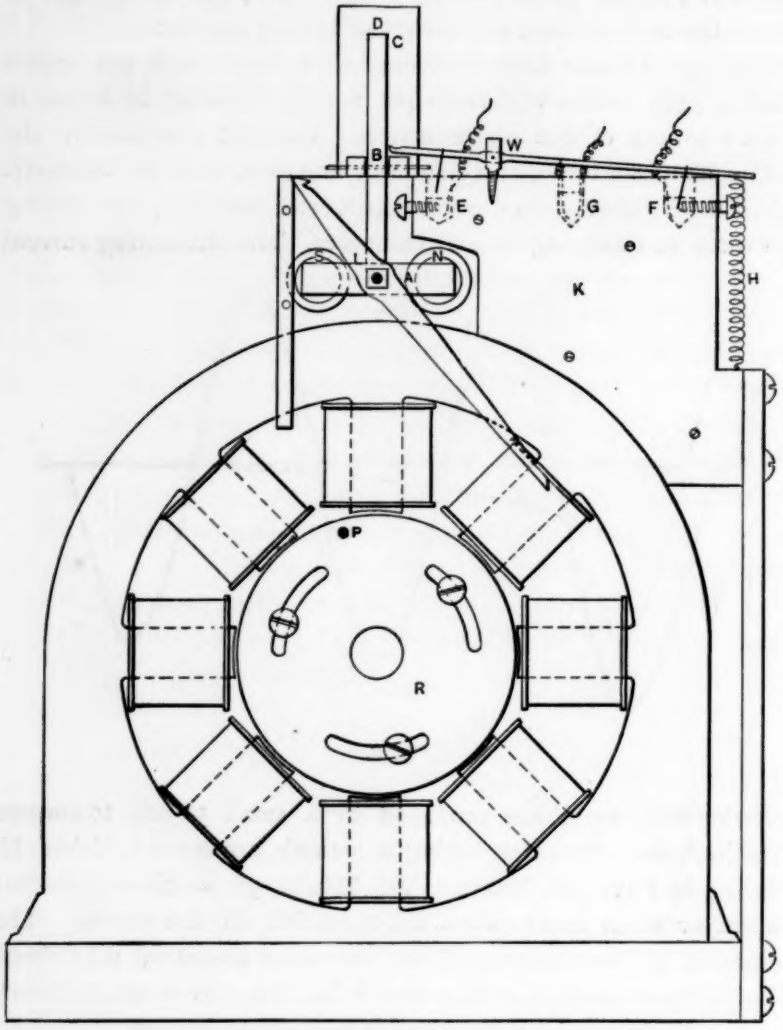

Fig. 3.

ends of the magnet being flush with the front surface of the plank. To the center of the magnet-armature *A* is rigidly fastened the brass upright *BC*, pivoted at *B*. The board *D*

is fastened to the back of the plank and a coiled wire spring attached to *D* and *C* (not shown) draws *C* back so that the armature is held away from *SN* when the magnet is not excited. The trip-lever *L* is pivoted to the center of the armature *A*. A board *K* is fastened to the front side of the plank, and in this the mercury cups *E*, *G*, and *F*, and the support for the stout wire *W* are placed that they may be in the same plane as the top of the lever *L*. *R* is a hard-wood board secured by three set screws to the end of the armature of the generator. *P* is an iron pin projecting about $\frac{3}{8}$ of an inch from the front surface of this board.

Now suppose the lever *L* to hang vertically, its top hooked over the end of *W*, lifting the contact wire out of the mercury cup *F* and making contact in the cup *E*. One pole of the alternator is connected directly to the galvanometer; the other pole is connected to the mercury cup *G*, and the circuit is completed through the cup *E*, the self-induction coil, and the galvanometer. The spring attached to *C* holds *A* away from the electromagnet so far that the end of the lever *L* is not struck by the pin *P* revolving with the armature of the machine. But as the photographic plate in falling passes the horizontal slit in its slide it closes a battery circuit (from 10 storage cells) through the electromagnet pulling *A* in so that *P* strikes *L*, and the dynamo circuit is suddenly broken at *E*, the spring *H* exerting a considerable pull.

To *close* the circuit which we are studying we have only to connect *F* instead of *E* into the circuit. As the armature of the alternator advances, an appreciable distance between the tripping of *L* and the actual break or make of the circuit at *E* and *F* respectively, the phase in which the break or make actually took place was determined by developing a few trial plates. *E* and *F* each had a thumb screw projecting into its side by which the surface of the mercury could be accurately adjusted through a small range in height. These with the set screws in *R* brought the phase of break and make under easy control.

To get a curve showing the dying away of a current when the electromotive force is suddenly reduced to zero, under the conditions discussed below, it is necessary to cut out the electromotive force and at the same time leave the remaining portion of

the circuit closed through the same resistance and self-induction as before. That is, the circuit must be closed through the self-induction and the galvanometer so as to include a new resistance equal to that cut out with the electromotive force, and this must be done at the *same instant* that the electromotive force is cut out.

To accomplish this, E is connected to one pole of the alternator (or to whatever source of current one wishes to study), G is connected to one circuit terminal, and the other circuit terminal is connected both to the dynamo and to F. The connection with F is through a resistance equal to that cut out with the electromotive force and of the same character.

The point now requiring care is to make the adjustments such that one wire shall leave the mercury at E at the *same instant* that the other touches the mercury at F. Adjustments must be made so that only a very small spark, and preferably none at all, shall be made at E, and yet connections at F must not be made soon enough to cause a short circuit. This is not difficult to accomplish, provided the mercury cups are large enough so that the wires shall not materially change the shape or the height of the surface of the mercury upon entering or leaving it. Figures 4, 5, and 6 show these different connections. A few make and break

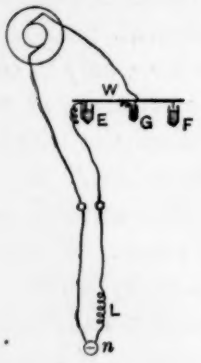

Fig. 4.
Break.

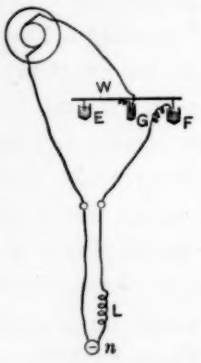

Fig. 5.
Make.

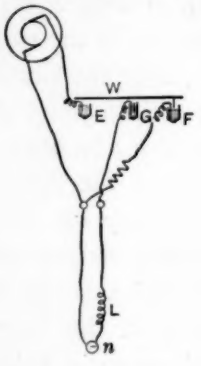

Fig. 6.
Cut Out.

curves from continuous currents were taken as a simple means of getting a satisfactory relation between the resistance and the self-induction in the circuit, and of making the necessary adjustments.

Let a continuous current flow in a circuit until it has reached its steady state. If the electromotive force be then suddenly cut out and the circuit at the same time closed in such a manner that the self-induction and the resistance remain the same, the general equation

$$e = Ri + L\frac{di}{dt} \tag{4}$$

becomes

$$o = Ri + L\frac{di}{dt}, \tag{5}$$

which integrates into

$$i = ce^{-\frac{Rt}{L}}. \tag{6}$$

Substituting for c its value determined by the condition that at the instant of cutting out the electromotive force i equals its steady value I, we get

$$i = Ie^{-\frac{Rt}{L}}.^{1} \tag{7}$$

From the same fundamental equation we get

$$i = I(1 - e^{-\frac{Rt}{L}})$$

or

$$I - i = Ie^{-\frac{Rt}{L}}, \tag{8}$$

as the equation according to which the current attains its steady value upon closing the circuit. Hence the curve along which the current picks up to its final value upon closing the circuit is of the same form as that along which the current dies down upon removing the electromotive force, except it is turned upside down and displaced vertically the height of its maximum ordinate. Figures 7 and 8 show one of each of these curves. When superposed, these curves coincide perfectly, except a slight difference at the very beginning due to a lack of perfect adjustment of the mercury in the cups E and F.

If the alternating electromotive force be removed when the phase angle is 90°, that is, when i has its maximum value I, we have

$$i = Ie^{-\frac{Rt}{L}}, \tag{9}$$

the same as when removing a constant electromotive force. Such a curve is shown in Fig. 8 of the article above referred to. By

[1] Alternating Currents, Bedell and Crehore, p. 45.

substituting for c in (1) its value given in (2), that term alone gives a curve of the form

$$i = -Ie^{-\frac{Rt}{L}}, \quad (\psi = 90°), \tag{10}$$

which is identical with the curve given by equation (8) except it is below the axis of reference. This curve cannot be obtained by itself on the photographic plate, as can all the others, because as soon as the circuit is closed the *alternating* current interferes with it, and the curve actually obtained is the combination of this logarithmic curve and the alternating current curve.

The combination of an alternating curve with another curve of any form whatever is in effect the same as making the alternating curve symmetrical about the second curve as an axis. Hence the curve obtained upon closing an alternating current circuit having self-induction and resistance is symmetrical about the logarithmic component of the curve. But this logarithmic component is the same as the curve of dying out, inverted and properly displaced. Hence if we obtain this curve of dying out on one plate, and on another plate obtain the combination curve produced by closing the circuit, and then invert the first and superpose them so that the same ordinate shall pass through the point of breaking the one and the closing of the other, the combination curve should be symmetrical about the logarithmic curve as its axis. Figures 9 and 10 show the two curves obtained separately, and Fig. 11 shows them superposed as above indicated. The straight line in Fig. 9 was made by dropping the plate when the needle was not deflected, and is merely to indicate the axis of the curve. The plate was dropped a second time to obtain the curve. In superposing the curves it was made to coincide with the straight line part of Fig. 10, which being the trace of the spot of light before the circuit was closed, is in the line of the axis of the curve after the distorting effect of the exponential term has become negligible. Careful measurements with the dividers do not show any lack of symmetry.

In a further communication the writer expects to discuss experiments made upon circuits containing capacity.

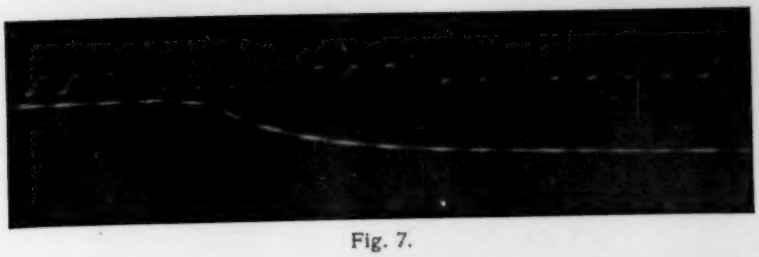

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

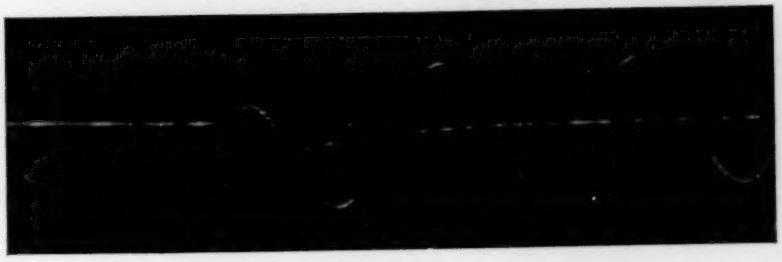

Fig. 11.

MICH.

DEMAGNETIZATION FACTORS FOR CYLINDRICAL RODS.

By C. Riborg Mann.

THE effect of the form of a substance on the intensity of magnetization which it assumes when brought into a magnetic field has been a much-discussed question. F. Neumann[1] attacked the problem from the analytical side, and showed that the intensity of magnetization can be calculated when the magnetization is uniform. He further demonstrated that the magnetization induced in any substance, when brought into a uniform field, will itself be uniform only when the substance is bounded by a surface of the second degree.[2] As the ellipsoid is the only surface of second degree whose dimensions are finite, it is the only form which is of practical interest to us here.

For the special case of a prolate spheroid brought into a uniform magnetic field whose lines of force run parallel to the axis of revolution of the spheroid, Neumann gives the following formula for calculating the intensity of magnetization :[3]

$$C = \kappa Z \frac{1}{1 + N\kappa};$$

Or in the modern notation,

$$\mathfrak{J} = \kappa \mathfrak{H} \frac{1}{1 + N\kappa}, \qquad (1)$$

in which $\mathfrak{J}$ = intensity of magnetization,

$\mathfrak{H}$ = strength of field,

κ = susceptibility,

N = a constant depending on the ratio of the axes of the ellipsoid.

[1] Neumann, Crelle's Journal, Vol. XXXVII., p. 44.

[2] Maxwell, Electricity and Magnetism, § 437.

[3] Maxwell, Electricity and Magnetism, § 438.

For our special case, the one of greatest practical use, this factor N is given by the following formula:

$$N = +4\pi\left(\frac{1}{\epsilon^2} - 1\right)\left(\frac{1}{2\epsilon}\log\frac{1+\epsilon}{1-\epsilon} - 1\right), \tag{2}$$

in which $\epsilon = \sqrt{1 - \frac{b^2}{a^2}}$, when b stands for the smaller, a for the greater semi-axis of the ellipsoid.

We note that N depends only on a and b, *i.e.* on the form of the ellipsoid in question.

If we clear the equation (1) of fractions, we get

$$\mathfrak{J} = \kappa\,(\mathfrak{H} - N\mathfrak{J}). \tag{3}$$

When κ is very small, as in all substances except nickel, cobalt, and iron, the coefficient of N, *i.e.* $\kappa\mathfrak{J}$ is very small in comparison to $\kappa\mathfrak{H}$, and is, therefore, generally neglected, thus making the determination of κ practically independent of the form of the material investigated, *i.e.* independent of N. Hence in what follows, as we are to discuss N, we will speak only of the paramagnetic substances where κ is large.

When in formula (2) $a = \infty$, or $e = 1$, *i.e.* when the ellipsoid considered is endless, $N = 0$, and formula (3) becomes $\mathfrak{J} = \kappa\mathfrak{H}$, the fundamental equation for the intensity of magnetization. As a grows smaller, b remaining constant, N increases. Hence this N in formula (3) characterizes the effect of the free ends of the ellipsoid on its own magnetization, and is called, since the action of the free ends in paramagnetic substances is to diminish the effective magnetic field, the demagnetization factor.

If now N is known, $\mathfrak{J}$ and $\mathfrak{H}$ being measurable, formula (3) may be used to calculate κ, the magnetic susceptibility of any substance. Hence we may determine κ either on rings, where $N = 0$, as Rowland did, or on ellipsoids, where N can be calculated from formula (2). Both of these methods are open to the practical objection that it is difficult to obtain either a suitable ring or an ellipsoid made of the material whose susceptibility we wish to know. Could we but make our observations on cylinders, which are easily procured, the task of determining κ would in most

cases be much facilitated. The following investigation was therefore undertaken to determine whether N is independent of $\mathfrak{B}$ and $\mathfrak{H}$ for cylinders as for ellipsoids, and whether or not its value is the same for a cylinder as for the corresponding ellipsoid, *i.e.* one whose greater axis is equal to the length, and whose smaller axis is equal to the diameter of the cylinder. The magnetization in the case of a cylinder is not uniform, as it is in ellipsoids; therefore, by the intensity of magnetization of a cylinder is understood a mean value obtained by dividing the total magnetic moment by the volume. Hence the magnetometric method was preferred for this work.

It was formerly assumed that a cylinder could be used for its corresponding ellipsoid of revolution in observations on magnetization by induction.[1] Du Bois,[2] on the other hand, has shown from observations of Ewing[3] and Tanakadaté,[4] that this is not the case. From these observations he deduces a table giving N for various values of the ratio $\frac{\text{length}}{\text{diameter}}$ of the cylinder, this ratio being denoted m. These values for N may be used to reduce observations made on cylinders to those made on corresponding ellipsoids or on rings.

The observations on which this table is based, though the best that then existed, are not entirely satisfactory for computing these factors, and this for several reasons.[5] *First*, those of Ewing were made with too short a magnetizing coil, so that his magnetizing fields were not uniform throughout the whole space occupied by his core. *Secondly*, the two sets do not join, as Ewing's shortest cylinder had $m = 50$, while for Tanakadaté's longest cylinder m was only 39, thus necessitating an extrapolation over the interval $39-50$. *Thirdly*, the two sets of measurements were made in different ways, Ewing having used the ballistic, Tanakadaté the magnetometric method. In my opinion the results from these

[1] W. Weber, Electrodynamische Maasbestimmungen, III., p. 573, 1867; Kirchhoff, Ges. Abh., p. 221; Oberbeck, Poggendorff's Annalen, CXXXV., p. 84, 1868.

[2] Magnetische Kreise, Berlin, 1895.

[3] Philosophical Transactions, 176, II., p. 535, 1885.

[4] Philosophical Magazine, 5 Series, Vol. XXVI., p. 450, 1888.

[5] *loc. cit.* p. 535.

two different methods of measuring induction cannot be used together with certainty in a case like this.

Ascoli,[1] however, has recently published a table of these factors obtained from observations on cylindrical bundles of iron wire, which agreed very closely with that of Du Bois. Nevertheless it seemed doubtful to me if these numbers of Ascoli's could be used with certainty for the demagnetization factors of solid cylinders, because bundles of wire, as has been noted already by several physicists,[2] do not react towards magnetization as do solid cylinders of the same material, length, and cross-section, which we may term corresponding cylinders.

In order now to solve the problem satisfactorily, I proceeded as follows: I made a long series of magnetization curves, using cylinders of the same material but of different form, using also bundles of wire, being exceedingly careful that the results should be strictly comparable with each other, and from these deduced the conclusions given below. To give a detailed list of the observations were to take far too much space. The method used was briefly this:[3]

A long thin soft-iron wire was taken, for which $m = 300$ (length 25.08 cm., diameter 0.0836 cm.). After determining the curve of magnetization, giving the valuation between $\frac{1}{3}$ and $\frac{2}{3}$, equal lengths were cut from each end, so that the wire assumed the form $m = 200$. The magnetization curve was again determined and the wire again shortened to $m = 150$, etc., until m became equal to 50. These observations were made after the usual magnetometric method, using a coil 38.5 cm. long of 1.5 cm. inner diameter, thus assuring me of a uniform field throughout the entire space occupied by the iron. The current was measured by a carefully calibrated ammeter and the strength of field calculated in the usual way by multiplying the number of amperes by the constant of the coil obtained by the well-known formula.

The effect of the magnetizing coil on the magnetometer was

[1] Rendiconti della royale Academia dei Lincei, 3, p. 190, 1894.

[2] v. Waltenhofen, Wiener Berichte, 61, II., p. 771, 1870; Warburg and Hönig, Wiedemann's Annalen, p. 828, 1883.

[3] The details of this work were published as a dissertation in Berlin, 1895, which may be obtained from the author.

balanced by another smaller coil through which the magnetizing current flowed in the opposite direction to that which it took in the main coil. This small coil was so placed that it just balanced the effect of the magnetizing coil on the magnetometer when the needle was at the zero point. Slight corrections had to be applied in many cases to the readings of the magnetometer when the needle was deflected, because this compensation was not perfect except at the zero point. Having adjusted this smaller coil so that the reading of the magnetometer remained the same when the current through both coils was flowing in either direction or

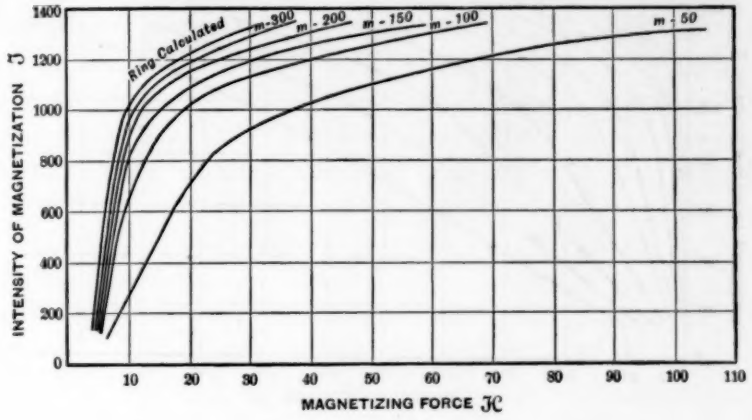

Fig. 1.

Magnetization Curves for Cylinders for Values of m between 300 and 50.

not flowing at all, the iron to be investigated was brought into the center of the magnetizing coil and subjected to a small field. The readings of both ammeter and magnetometer were taken and the polarity of the field was reversed, its strength remaining as nearly constant as possible, and then a second reading was taken. The means of these two opposite readings were used to calculate, by the usual formula,[1] the corresponding values of $\mathfrak{H}$ and $\mathfrak{J}$. The field was then strengthened and the same operation repeated, — in short, the method of ascending reversals was used. From these cylinders made of iron wire, the magnetization curves in Fig. 1 for values of m between 300 and 50 were made.

[1] Wiedemann, Electricität, 3, § 428.

For shorter cylinders a different method was used. A short, thick rod for which m was only 5 (length, 11.850 cm.; diameter, 2.370 cm.), was gradually turned down, the length remaining constant, till m reached the value 50; *i.e.* till its form was the same as that of the shortest of the former set. Thus the two sets joined together, and in each the same iron was used throughout. This second set ($m=5$ to $m=50$) was made twice, using different qualities of iron and a different length.[1]

The first set was executed three times and the mean taken. Each curve of the other set was run at least three times and the

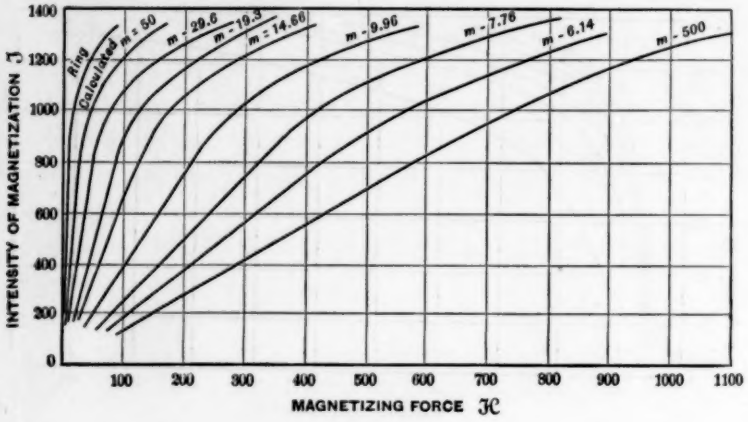

Fig. 2.

Magnetization Curves for the First Cylinder.

mean taken. The curves given in Fig. 2 are these mean curves. To get the demagnetization factors N from the curves, we proceed as follows: Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be values of $\mathfrak{B}$ which belong to the same $\mathfrak{H}$ on any two curves, and N_1 and N_2 the corresponding values of N. Then from (3)

$$\mathfrak{H}=\kappa(\mathfrak{B}_1-N_1\mathfrak{H})=\kappa(\mathfrak{B}_2-N_2\mathfrak{H}),$$

or

$$N_2=N_1+\frac{\mathfrak{B}_2-\mathfrak{B}_1}{\mathfrak{H}}; \qquad (4)$$

i.e. we must measure the difference in $\mathfrak{B}$ of the two curves under consideration along the same $\mathfrak{H}$-line, and divide this difference

[1] For the second cylinder the length was 9.620 cm.

by the $\frac{1}{m}$ which belongs to that line, and add the quotient thus obtained to the demagnetization factor corresponding to the first curve.

This method presupposes the knowledge of N_1. There are several methods by which this factor can be determined. In my table below I assumed the curve for which $m = 300$ as the curve 1. The value of the corresponding N, which I will designate N_{300}, I unfortunately did not have time to determine by an independent method before the work was of necessity broken off. I assumed $N_{300} = 0.00075$, the value belonging to the corresponding ellipsoid, and for the following reason: Du Bois has shown theoretically that, for cylinders whose length is very much greater than their diameters, the quantity Nm^2 should be constant. From Ewing's observations he deduces the value of this constant as 45. If I assume this theoretical law of du Bois, I have the condition necessary to determine N_{300} from my observations as du Bois did.[1] The work is as follows, using the data from my observations:

m	N	x	Nm^2
300	x		
200	$0.00079 + x$	0.00041	48.0 (200 and 150)
150	$0.00172 + x$	0.00041	48.0 (200 and 100)
100	$0.00438 + x$	0.00041	47.9 (150 and 100)
Mean		0.00041	48.0

It is quite evident that my observations do not satisfy the theoretical conclusions of du Bois. A similar calculation for ellipsoids gives:

m	N	x	Nm^2
300	x		
200	$0.00085 + x$	0.00044	51.6 (200 and 150)
150	$0.00185 + x$	0.00042	50.8 (200 and 100)
100	$0.00465 + x$	0.00040	50.0 (150 and 100)
Mean		0.00042	50.8

[1] Wiedemann's Annalen, XLVI., 1892.

These two tables are seen to be very similar. Therefore it is very probable that these long cylinders act very similar to their corresponding ellipsoids, as has always been assumed;[1] and hence I felt warranted in assuming the value I did for N_{300}.[2]

Having this for a starting point, the values of N for the other curves are easily calculated by formula 4.

The measurements on bundles of wire were conducted simultaneously with those on the second short cylinder. The wire used was 0.0801 cm. in thickness, and cut into lengths of 9.8 cm.

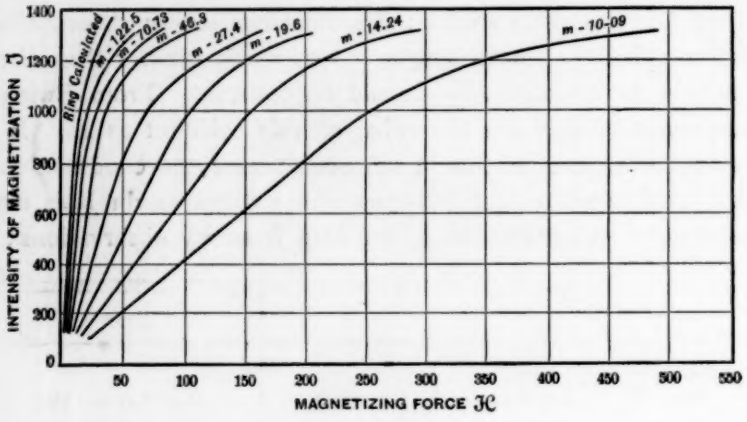

Fig. 3.

Magnetization Curves for Wire Bundles.

These lengths were bound into cylindrical bundles, and the magnetization curves (given in Fig. 3) determined as for solid cylinders.

The size of the bundles varied from a single wire, for which $m=122.5$, to 171 wires, for which $m=9.37$. The value of N_1 for the wire bundles was of course that for a cylinder of the form of a single wire; *i.e.* the value corresponding to $m=122.5$. This factor was taken from the table of N for solid cylinders. In interpolating and comparing the observations, the factor N alone was not used, but rather the expression Nm^2, as this latter serves much better for this purpose. Figure 4 contains the curves

[1] Maxwell, Electricity and Magnetism, § 438.

[2] It is probable that it should be a trifle smaller, say 0.00070; but this difference of 0.00005 is not appreciable for the shorter cylinders.

$Nm^2 = f(m)$ for solid cylinders, ovoids, and wire bundles. The points represent the various observations.

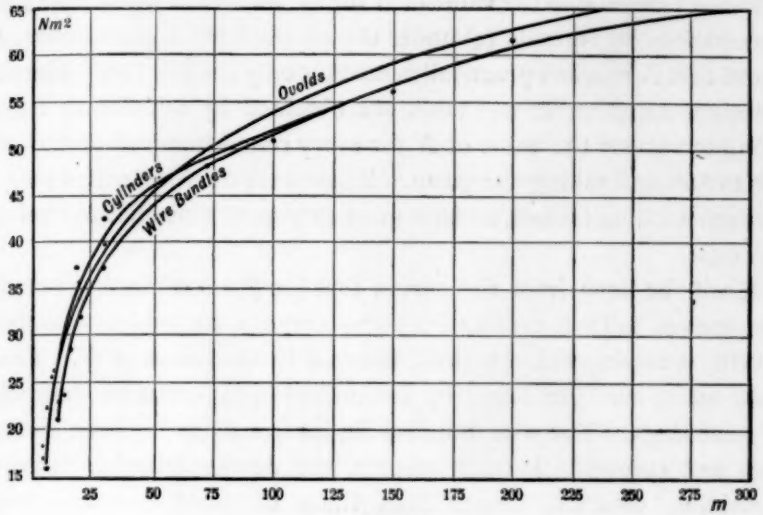

Fig. 4.

Nm^2 as a function of m.

The following table gives corresponding values of m, N, and Nm^2 for the three cases:

	Cylinders.		Ovoids.		Wire Bundles.	
m	N	Nm^2	N	Nm^2	N	Nm^2
5	0.68000	17.0	0.7015	17.5	—	—
10	0.25500	25.5	0.2549	25.5	0.22750	22.8
15	0.14000	31.5	0.1350	30.4	0.12580	28.3
20	0.08975	35.9	0.0848	34.0	0.08225	52.5
25	0.06278	39.3	0.0579	36.2	0.05680	35.5
30	0.04604	41.4	0.0432	38.8	0.04213	37.9
40	0.02744	43.9	0.0266	42.5	0.02596	41.5
50	0.01825	45.6	0.0181	45.3	0.01760	44.0
60	0.01311	47.2	0.0132	47.5	0.01277	46.0
70	0.00988	48.4	0.0101	49.5	0.00951	47.8
80	0.00776	49.7	0.0080	51.2	0.00768	49.1
90	0.00628	50.8	0.0065	52.5	0.00623	50.5
100	0.00518	51.8	0.0054	54.0	0.00515	51.5
150	0.00251	56.5	0.0026	58.3	—	—
200	0.00152	60.8	0.0016	64.0	—	—
300	0.00075	67.5	0.00075	67.5	—	—

The column N for ovoids was calculated from formula (2). They are good for all values of $\mathfrak{H}$.

When I calculated the value of N for cylinders and wires from the observations by formula (4) under the supposition $N_{300}=0.00075$, I found that N remains practically constant only for $\mathfrak{H}<800$. Hence the numbers given in the table are obtained by calculating from the observations the value of N for every round hundred of $\mathfrak{H}$ from 300 to 800, and taking the mean. They are, therefore, called mean demagnetization factors, and are good only until $\mathfrak{H}$ reaches the value 800 c.g.s.

It will be seen from the curves that for $\mathfrak{H}>800$ the magnetization curves fall off rapidly from the curve 1 for which $m=300$, causing a correspondingly rapid increase in the values of N. This same effect has been noted by Lehmann[1] in experiments on radially cut rings. The wire bundles lie intermediate between cylinders and ellipsoids in this respect, the demagnetization factors remaining constant longer than those of their corresponding cylinders.

The following table for $\mathfrak{H}$, with the corresponding value of N, will illustrate the point in hand:

$\mathfrak{H}$	N		$\mathfrak{H}$	N	
	m = 46.30 wires.	m = 29.60 cylinders.		m = 46.30 wires.	m = 29.60 cylinders.
300	0.02167	0.04718	900	0.01950	0.05166
400	0.01907	0.04822	1000	0.02085	0.05877
500	0.02024	0.04873	1100	0.02480	0.07964
600	0.01984	0.04870	1200	0.03287	0.09768
700	0.01976	0.04892	1300	0.05181	0.12777
800	0.01962	0.04735			

The results may be summed up as follows.

The mean magnetization of a cylinder does not differ greatly in amount from the magnetization of the corresponding ellipsoid for values of $\mathfrak{H}<800$ c.g.s. For $\mathfrak{H}>800$ an ellipsoid assumes a much stronger magnetization for the same magnetizing force than its corresponding cylinder.

[1] Wiedemann's Annalen, XLVIII., p. 406, 1893.

Wire bundles assume, when $\mathfrak{H} < 800$, a much stronger magnetization for the same magnetizing force than either their corresponding ellipsoids or cylinders. The ellipsoid has, however, greater susceptibility for higher values of $\mathfrak{H}$.

Cylinders whose length is from 20 to 30 times their diameter differ most from the corresponding ellipsoids in their reaction towards induced magnetization.[1]

These values of N for cylinders, when used in formula (3), will give the correct value of κ, provided only that $\mathfrak{H} < 800$ c.g.s.

This result is practically of value, as it enables us to determine κ from observations made by the ordinary magnetometric method on cylinders.

The above investigation was carried on in the physical laboratory of the Berlin University, under the direction of the late Professor Kundt and Professor Warburg, whose kindness and assistance I wish here gratefully to acknowledge.

UNIVERSITY OF CHICAGO, January, 1896.

[1] Cf. Tanakadaté, Philosophical Magazine, 5 Series, Vol. XXVI., p. 453, 1888.

A PHOTOGRAPHIC STUDY OF ARC SPECTRA. I.

BY CAROLINE WILLARD BALDWIN.

INTRODUCTION.

IN the course of his investigation of the Infra-Red Spectra of the Alkalies, Professor Snow[1] has shown the remarkable effect produced upon the arc spectrum when the alkalies were present in the carbons. He found that the curve obtained from his bolometric measurements was materially changed by the metals. While in the ordinary arc he had several very strong maxima in the ultra-violet and in the visible spectrum, he discovered that upon introducing the metals, these intense regions disappeared and strong maxima were now observed only in the infra-red.

The maxima in the violet and ultra-violet are the well-known characteristic, bright groups of the arc spectrum. These are produced by a peculiar crowding together of fine lines in the regions $\lambda = 3450$ to $\lambda = 3590$, $\lambda = 3700$ to $\lambda = 3885$, and $\lambda = 4030$ to $\lambda = 4211$. The disappearance of these and other similar groups would amount to a practical obliteration of the arc spectrum and substitution in its stead of the line spectrum of the particular metal used.

The phenomena seemed worthy of further study, and as the bright groups lie in the regions to which the photographic plate is most sensitive, it was thought worth while to try the application of photography to determine whether the disappearance noted was complete, or whether the bright regions were only so much reduced in intensity as to escape detection by bolometric observation.

As a matter of fact, the work was extended beyond its original limit, and in the end it assumed the form of a photographic study of the spectrum obtained from different regions of the arc under

[1] B. W. Snow, PHYSICAL REVIEW, Vol. I., p. 28.

different conditions, and of the effect of the metallic spectra upon the original carbon arc spectrum.

Apparatus.

The spectrum to be photographed was produced by means of a Rowland concave grating, which was arranged in the usual way upon a Brashear mounting. The latter consisted of a strong iron frame, which carried two tracks, *SG* and *SC*, Fig. 1, at right angles to one another. At their point of intersection *S*, the slit was placed. The carriage *G*, which contained the grating, moved on one track, and the carriage *C*, for the camera box, on the other. These two carriages were connected by an iron rod to which they

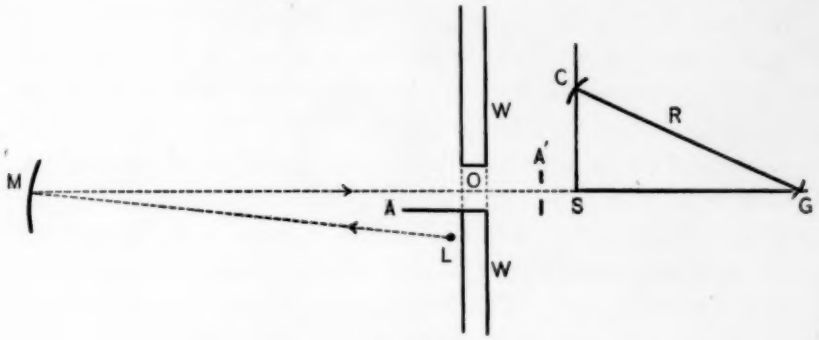

Fig. 1.

were clamped in such a position as to keep the rod in the common normal to the grating and photographic plate. A screw on the camera box permitted the finer adjustments of position to be made.

The width of the slit was adjusted by means of a micrometer screw, 0.1 mm. being the width ordinarily used. The length of the slit was gauged by means of a diaphragm having a wedge-shaped opening. A frame which held the slit could be turned by means of a screw and spring so as to place the slit accurately parallel to the lines of the grating.

The grating was of six-foot radius, and 14,000 lines to the inch.

Celluloid films were used, and, as these could be readily bent to

any curvature, the film holder was curved to conform to the focus of the concave grating. A fence work of horizontal and vertical bars placed back of the film kept it from curving longitudinally, and yet permitted the spectrum to be seen from the back, so that a cleared film could be used for focussing.

The light used was obtained from a Thomson and Houston arc lamp, which was hung so that the arc could be kept nearly in the horizontal plane in which lay the center of the grating and the central points of the slit and camera box. The carbons were 1. cm. in diameter, unplated, and were made with a soft core .3 cm. in diameter. To produce the arc spectra of the metals, the cores were removed and the space was filled with the metallic salt well pounded in.

In order to prevent fogging of the photographic plates, the lamp *L*, Fig. 1, was placed in the outer of two dark rooms, and the light was reflected into the inner room, which contained the grating, by means of a large concave mirror (*M*). This mirror was mounted so as to be adjustable about both a horizontal and a vertical axis. Its radius of curvature was twelve feet. The arc light was placed nearly at the center of curvature of the mirror, which was so adjusted that a real, slightly enlarged image was formed on the slit (*S*). This arrangement was also of advantage, inasmuch as it did away with the use of a converging lens. The arrangement of the apparatus is given in Fig. 1, to which reference has already been made.

The opening between the two rooms was closed on the outer side by a slide provided with a circular opening, and on the inner side by a solid slide, by means of which the light could be entirely shut out.

A screen near the lamp cut off the direct rays of the opening between the rooms; and another screen was placed a short distance in front of the slit, to intercept the scattered light.

OUTLINE OF THE WORK.

The image of the arc, as examined by the eye, shows sheaths of different colors. The arc is slightly cone-shaped, the apex

resting on the negative carbon. The central portion (1), which is of a violet tint, connects the two bright points of the carbons. Outside of this is a sheath of dull blue (2), which is most brilliant at the negative carbon, where it extends across in such a way as to hide the violet portion. The outer sheath (3) is yellow, shading into orange at the cooler outer edge; this last part includes the greater portion of the flame of the arc, and extends well up around the positive carbon. The different divisions are shown in Fig. 2.

Fig. 2.

The investigation may be considered as divided into four parts:

I. Photographs were taken with the slit extending vertically through the arc in six different positions: first, through the center of the violet portion; second, at the line of separation between

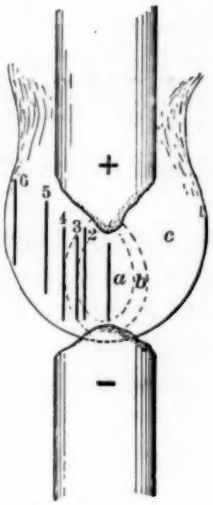

Fig. 3.

Fig. 4.

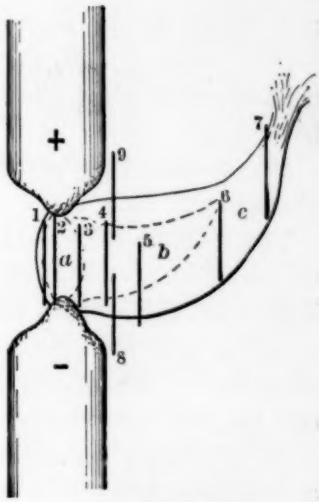

Fig. 5.

the violet and blue; third, in the blue; fourth, between the blue and the yellow; fifth, in the yellow; and sixth, at the extreme outer edge of the yellow (see Fig. 3).

II. Photographs were also taken with the slit extending horizontally through the image of the arc. Three positions were chosen

for these: first, near the tip of the negative carbon; second, half-way between the carbons; and third, near the positive carbon.

III. The flame was blown out from between the carbons by means of a horseshoe magnet. Under these circumstances the violet part does not change much in its position, but is slightly extended on the side away from the magnet; while the blue and yellow are blown out nearly three centimeters beyond the edge of the carbons. The appearance of the arc was as shown in Fig. 4. Photographs were taken with the slit extending vertically through this image in nine different places (Fig. 5): first, at the inner edge of the arc; second, in the center between the tips of the carbons; third, in the outer part of the violet; fourth, at the edge of the carbons; fifth, outside of the carbons through the blue and yellow; sixth, in the yellow; seventh, at the extreme end of the flame; eighth and ninth, at the edge of the carbons, giving the extent of the flame in the first place at the side toward the negative carbon, and in the second place at the positive carbon.

One hundred and thirty photographs were taken by these methods, from which the study of the ordinary arc spectrum has been made.

IV. Metals were introduced into the arc, and similar sets of photographs were taken.

The metallic salts used were lithium carbonate, sodium nitrate and chloride, potassium chloride, calcium chloride, strontium oxide, barium chloride, copper sulphate, silver nitrate, zinc chloride, and cadmium chloride. Sodium and zinc were studied by all three methods, the others only by the first two. This series included one hundred and fifty photographs.

All of these photographs were taken in the primary spectrum, as there was less confusion due to overlapping of spectra. The ultra-violet of the second spectrum, which extended to the green of the first, could be easily cut off by means of a glass plate placed in front of the slit. This gave a pure spectrum in the region to which the plates were sensitive.

In order to have the negatives evenly exposed, and the intensity as nearly as possible the same for different parts of the spectrum, four positions were chosen for photographing. *First*, the ex-

treme ultra-violet, including the first bright group ($\lambda = 2263$ to $\lambda = 3600$); *second*, the region of the three principal bright groups ($\lambda = 3450$ to $\lambda = 4211$); *third*, from the brightest of the groups to the *D* lines, or from $\lambda = 3800$ to $\lambda = 5900$; and *fourth*, with a very long exposure, the region between $\lambda = 4500$ and $\lambda = 6400$. The exposures varied in length from thirty seconds in the violet to five or ten minutes in the extreme ultra-violet, and fifteen to twenty minutes in the red of the spectrum.

On account of the motion of the arc it was necessary to have a movable screen, by means of which the light could be cut off whenever the arc moved so as to throw the wrong part of the image upon the slit. This movement of the arc caused so much trouble that it was difficult to obtain good photographs of the different sheaths in the regions of the spectrum which required long exposure. Hence the study of the spectra of different parts of the arc has been confined to the region between $\lambda = 3010$ and $\lambda = 5500$; and toward the ends of this region photographs were taken only in the violet, blue, and yellow sheaths of the image. A slightly longer exposure was given in the outer part of the arc than was needed in the violet and blue portions.

The length of the arc used was about 1.4 cm.

I.

Results obtained with the Ordinary Arc.

The distinctive features of the arc spectrum are the bright groups[1] from $\lambda = 3520$ to $\lambda = 3590$, $\lambda = 3800$ to $\lambda = 3885$ mentioned by Kayser and Runge, Drude and Nunst, Nichols and Franklin, etc.; one from $\lambda = 4400$ to $\lambda = 4604$; the carbon bands[2] given by Kayser and Runge and others, $\lambda = 4680$ to $\lambda = 4737$, $\lambda = 4746$ to $\lambda = 5165$, and $\lambda = 5530$ to $\lambda = 5635$; together with a host of periodically placed fine lines. The bright groups appear to be due to a crowding together of these numerous fine lines. The strong lines of the various metals, which are present

[1] Snow, Physical Review, Vol. I., p. 28; Kayser and Runge, Wied. Ann., XXXVIII., p. 81, 1889; Drude and Nunst, Wied. Ann., XLV., p. 460, 1892; Nichols and Franklin, Am. J. (3), p. 106, 1889.

[2] Kayser and Runge, Ueber die Spectren der Elemente, Zweiter Abschnitt.

as impurities in the carbons, are superposed upon this under spectrum. The arc spectrum thus seems to consist of two spectra.

The general arrangement of the periodic lines is worthy of note. They are finer and nearer together toward the violet end of the spectrum; the bright groups are also nearer together in the regions of greater refrangibility. In each group, however, the lines are finer and nearer together toward the red of the spectrum, the maximum being toward the longer wave lengths, on which side the termination of the group is sudden. Each of the groups shows secondary maxima, and between each of these the same general law is followed, as in the case of the group as a whole. We shall speak of these groups hereafter under the general name of the band spectrum.

Many of the metallic lines are stronger near the negative carbon and are weak or invisible at the positive carbon,[1] while the lines of the band spectrum are strongest at the positive carbon.

As we pass from the center of the arc to the edge, the band spectrum grows fainter and finally disappears. This is also true of many of the metallic lines, while other metallic lines seem to be equally bright in all parts of the arc, and a considerable number are relatively stronger in the outer sheath. This effect is enough to change the entire aspect of the spectrum in many places. The lines which show the last effect do not belong exclusively to any one metal, neither do all the lines of a metal seem to act thus. In the case of the calcium lines, all the triplets of the 2d series given by Kayser and Runge[2] are strongest in the center, while the pairs

$$\left.\begin{matrix}3968.63\\3933.83\end{matrix}\right\} \text{ and } \left.\begin{matrix}3737.08\\3706.18\end{matrix}\right\}$$

and certain other lines as $\lambda=4226.91$ are much stronger in the yellow part of the arc flame. A few of the triplets of the first series are stronger toward the edge. Such are

$$\left.\begin{matrix}4318.80\\4302.68\\4299.14\end{matrix}\right\} \text{ and } \left.\begin{matrix}3644.45\\3630.82\\3624.15\end{matrix}\right\}$$

[1] Lockyer, Proc. Roy. Soc., XXVIII., 1879.

[2] Kayser and Runge, Ueber die Spectren der Elemente.

In the carbon band, ending at $\lambda=4735$, the maxima remain the same as we pass from center to edge of the flame, but the law seems to be changed. In the outer sheath the maximum intensity is toward the violet and the strongest maximum is $\lambda=4645$. The shading of the band is thus completely reversed. Another band with maxima toward the violet is seen in the outer sheath from $\lambda=4840$; the maxima being $\lambda=4840$, $\lambda=4865$ and $\lambda=4892$. The strongest part of the group at 3590 becomes the weakest in the outer part of the arc, and many groups have the faintest lines at the center, the strongest at the edge.

When the photographs are taken giving the spectrum of the flame as blown out from between the carbons by the action of a magnet, the slit extends across the different sheaths of the arc, and those lines which are stronger in the outer parts of the arc reach through the spectrum with their brightness undiminished or increased at the edges, while the bands and many of the metallic lines appear only in the central part.

When the slit is arranged to pass through the image of the arc horizontally, this effect is more marked, owing to the greater steadiness of the flame. In watching the developing of the negatives, it was observed that many of the lines, as 4226, 3583, 3571, were first visible at the edge and then gradually extended toward the center, while the fine lines of the band spectra and most of the other lines appeared first at the center and then extended part way to the edge. The photographs taken by the three methods thus show complete agreement in their results.

Since the spectrum of the arc has already been mapped with more powerful apparatus than that used in these experiments, only such points will be given as may serve to indicate the differences between the spectra of the different sheaths.

(A) *Region lying between* 2263 *and* 3100.

135 lines were mapped between 2263 and 3016, all of which belonged to the central violet sheath [(1) Fig. 2].

The first lines which could be detected in the blue and yellow sheaths lay between 3018 and 3099; eleven lines among the nineteen mapped were visible in the blue and four in the yellow.

(B) *Region lying between* 3100 *and* 3440.

72 lines were mapped, all of which were confined to the central blue sheath.

(C) *Region lying between* 3440 *and* 3928.

135 lines were mapped, of which

125 were present in the violet,
96 were present in the blue,
58 were present in the yellow.

Of those which were invisible or very faint in the central violet sheath, although strong in the outer sheaths, the following are readily placed in Rowland's Standard List:[1]

3812.12, 3812.20, 3812.34 C.	3832.45 Mg.
3815.98 Fe.	3834.36 Fe.
3820.59 FeC.	3838.43 Mg.
3827.03 Fe.	3841.19 FeMn.
	3856.52 Fe.
	3860.05 FeC.

(D) *Region lying between* 3930 *and* 4550.

Throughout this region photographs were taken in the five positions indicated in Fig. 3, as *a*, 2, 3, 5, and 6.

213 lines were mapped, of which

211 lines were visible in central violet sheath (*a*),
200 lines were visible in outer violet sheath (2),
179 lines were visible in blue sheath (3),
175 lines were visible in inner yellow sheath (5),
57 lines were visible in outer yellow sheath (6).

The two lines absent from the central violet appear to be

3960.05 Fe, 4076.96 Fe } Rowland's scale.

[1] Rowland, Astrophysical Journal, I., p. 29, 1895.

(E) *Region lying between* 4550 *and* 4960.

Photographs were taken in three regions (1, 2, and 3, Fig. 2). 129 lines were mapped, of which

129 lines were visible in the violet sheath,
103 lines were visible in the blue sheath,
95 lines were visible in the yellow sheath.

(F) *Region lying between* 4960 *and* 6042.

115 lines were mapped, of which

115 lines were visible in the violet sheath,
42 lines were visible in the blue sheath.

If we sum up these data, we find

Total lines mapped	799,[1]
Visible in central violet	787,
Visible in blue	431,
Visible in yellow	232.

Attempts to classify these lines according to their sources led to no satisfactory results. A study of the photographs, however, showed that the lines of the arc spectrum may be grouped as follows:

I.*a*. Lines which grow gradually weaker as we move from the center to the edge, and are usually invisible in the outer sheath.

I.*b*. Lines which preserve about the same intensity in the center and blue of the arc, but are suddenly much weaker or invisible in the yellow.

Forty lines of this class were counted. They were distributed throughout the spectrum from 3024 to 5616.

The lines of the band spectra grow fainter through the blue, but disappear quite suddenly there, not being seen in the yellow.

I.*c*. Faint lines which are only found in the first two positions (*a* and 2, Fig. 3). Invisible in the blue. One hundred and sixteen of these were counted.

[1] Exclusive of the fine lines of the band spectrum which were not counted.

II. *a*. Lines which appear equally strong in all parts of the arc. Seventy such lines were counted.

II. *b*. Lines which become stronger as we go out from the center. Seventy-nine of these lines were mapped.

II. *c*. Lines which are not visible or exceedingly faint at the center of the arc. There are eighteen of these, of which exact positions for twelve have been given in a previous paragraph.

II. *d*. Lines which have their maxima in other sheaths, 2 or 3, rather than in the center or at the edge of the arc. There are sixteen such lines upon the photographs.[1]

II. *e*. Lines which have more than one maximum. Six such lines were noted.

The lines of division I. are as a rule *stronger at the positive carbon*, while the lines of II. are usually *stronger at the negative carbon*. II. *b*. and II. *c*. are especially strengthened at the negative carbon, II. *a*. to a less extent.

The calcium lines 4581.66 and 4586.12 appear as single lines at the negative carbon, but at the positive they seem to be double.

[1] There is reason to think that at least ten of these are titanium lines, but the evidence is not conclusive.

(To be concluded.)

MINOR CONTRIBUTIONS.

The Surface Tension of Liquids.

By Arthur L. Foley.

In the *Philosophical Magazine* of November, 1893, Mr. T. Proctor Hall describes some "New Methods of Measuring the Surface Tension of Liquids." Two years ago, at the suggestion of Professor Michelson of the Chicago University, I undertook to repeat and to extend the investigation. In the present article, I shall confine myself to a brief statement of the results obtained by using Mr. Hall's method *c*, the maximum-weight method.[1]

Let *a* (Fig. 1) be an end face of a rectangular parallelopiped suspended from one arm of a balance, with its lower face horizontal, and therefore parallel to the liquid surface *Ox*. Call w' the weight of the frame (block) in this position. Lower the frame until it touches the liquid, and bring it again to the first position, as in *b*. The weight of the frame is now increased by the weight of the liquid raised above the level surface. As the frame is raised, the weight increases for a time, then suddenly decreases, passing through a distinct maximum. Call w'' the total maximum weight. The net maximum weight is

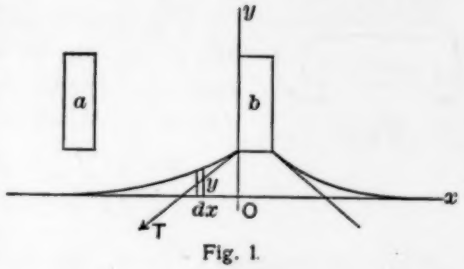

Fig. 1.

$$w = w'' - w' = 2T \sin \alpha + \rho t y, \qquad (1)$$

where T = the surface tension in grams per centimeter;
α = the angle between the X-axis and the tangent to the liquid surface at the edge of the frame;
t = the thickness of the frame;
ρ = the density of the liquid;
y = the height of the frame above the liquid surface;
l = the length of the frame, one centimeter.

[1] Philosophical Magazine, November, 1893, p. 402.

Also,
$$T \sin \alpha = \rho \int_{-\infty}^{0} y dx, \tag{2}$$

$$\frac{dx}{d\alpha} = \frac{T \cos \alpha}{\rho y}.$$

Placing $c^2 = \frac{T}{\rho}$, and remembering that $\frac{dy}{dx} = \tan \alpha$,

$$\frac{dy}{d\alpha} = \frac{c^2 \sin \alpha}{y};$$

$$y^2 = -2c^2 \cos \alpha + k.$$

When $y = 0$, $\alpha = 0$, and $k = 2c^2$.

$$\therefore\ y = 2c\sqrt{\frac{1 - \cos \alpha}{2}} = 2c \sin \frac{\alpha}{2}; \tag{3}$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{4c^2 - y^2}{4c^2}}; \tag{4}$$

$$\cos \alpha = \frac{2c^2 - y^2}{2c^2}. \tag{5}$$

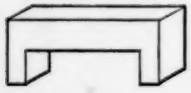

Fig. 2.

Let us now suppose that the frame has vertical legs (as in Fig. 2) extending downward into the liquid. Let l be the length between the legs.

Equation (1) becomes

$$w = 2T(l - t) \sin \alpha + \rho t l y, \tag{6}$$
$$= 2\rho c^2 (l - t) \sin \alpha + 2 l t \rho c \sin \frac{\alpha}{2}.$$

When w is a maximum, $\frac{dw}{d\alpha} = 0$. Let t be very small compared with l, then

$$2c \cos \alpha + t \cos \frac{\alpha}{2} = 0.$$

Eliminating α by (4) and (5), and inserting the value of c,

$$y = \sqrt{\frac{2T}{\rho} - \frac{t^2}{8}} - t\sqrt{\frac{T}{2\rho} + \frac{t^2}{64}}.$$

When t is small, a near approximation is

$$y = \sqrt{\frac{2T}{\rho}}. \tag{7}$$

Supplying this value of y in (6), and solving for T,

$$T = \frac{w}{2(l - t)} + \frac{\rho l^2 t^2}{4(l - t)^2} - \frac{lt}{4(l - t)^2}\sqrt{\rho^2 l^2 t^2 + 4w(l - t)\rho}. \tag{8}$$

Table II. gives the value of T calculated by the above formula for mica frames varying in thickness from 0.0013 cm. to 0.02067 cm.

Mr. Hall, in his investigation, used glass frames (made of cylindrical glass rods) of the shape indicated in Fig. 3. He deduced for them equations corresponding to (6), (7), and (8). He admits, however, that these equations are so complicated as to be almost unmanageable, and that the correction is obtained more easily by determining the constants of a frame by using frames of different length and of the same diameter, and again, of the same length but of different diameters. It is very difficult indeed to make such frames, and to use them after they are made.

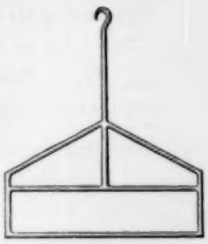

Fig. 3.

The chief objections to glass frames may be summed up as follows: —

The value of y, and hence the correction that must be applied to the maximum weight in order to obtain the true film weight which measures the tension, depends in a very complicated way upon the diameters of the rods of the frame.

This correction forms a considerable part of the total maximum weight (see Table I.). Frames cannot be made sufficiently rigid and less than 0.03 cm. in diameter. Hence the correction is at least 10 per cent of the whole.

The frames are difficult to make and they require delicate handling at every stage.

With cylindrical end rods the actual length of the film surface is uncertain.

It occurred to me that these troublesome corrections and inaccuracies might be partially avoided by using a different kind of frame. After experimenting with frames of various materials, among which I may mention thin sheet glass, platinum, aluminum, and mica, I found that the latter offered decided advantages over glass. The general shape of the mica frame is given in Fig. 4. The frame is supported by a forked glass stem, and the method of using is exactly as with a glass frame.

Fig. 4.

My first frames were made by cutting the mica sheet as it lay under a steel rule upon a piece of plate glass. I afterwards had made two heavy steel plates of the exact shape of the frame desired. The inner surface of each plate was ground plane with emery dust upon plate glass. A sheet

of mica was clamped between them and cut to their dimensions. The advantages of frames made in this way are:

The steel plates are accurately ground; the frames are correspondingly regular.

The mica does not split along the cut edge.

The edge is of the same thickness as the plate itself; there is no bur. Very thin frames are easily made, but it is difficult to work with them when they are much less than 0.002 cm. thick.

A difficulty experienced with the mica frame, as also with those of platinum and aluminum, is that the fluid does not readily and equally wet all portions of the surface. It has a tendency to collect in drops, rendering the after-weighing uncertain. This difficulty was entirely overcome by roughing the surface (darkened in Fig. 4) of the plate near the edge by rubbing very lightly with the finest French emery paper. Both weights could then be taken again and again with a variation of only a few hundredths of a milligram.

The advantages claimed for the mica frame are as follows:

1. They are easily made, and do not require careful handling.

2. They are of even thickness, with straight edges and square corners. Hence the film length is not so uncertain as with glass frames.

3. They can be made less than one-tenth of the thickness of a glass frame, reducing the correction correspondingly. Table I. gives the relative corrections for glass and mica frames, obtained by determining the maximum weight for a soap solution, and then weighing the film itself. The film weight divided by twice the length of the frame gives the surface-tension. But with many liquids it is impossible to obtain the film weight, as the film breaks immediately after it is formed. The maximum weight can be determined in almost every case, and the film weight by correction. It is evident that a slight error in the value of this correction will be lessened by reducing the total correction, as is done by using the mica frame.

TABLE I.

Kind of frame.	l	t	w	Film weight.	Per cent difference.
Glass . . .	6.346	0.0405	0.39226	0.34100	15
Glass . . .	7.584	0.0510	0.48283	0.40302	19
Glass . . .	10.163	0.0620	0.65420	0.53700	21
Glass . . .	7.475	0.0920	0.52480	0.39660	32
Mica . . .	6.012	0.0030	0.31202	0.30697	1.6
Mica . . .	5.301	0.0051	0.27776	0.27092	2.5
Mica . . .	5.140	0.0079	0.27222	0.26260	3.7

A fresh solution was used in the last three measurements.

4. The correction varies directly as the thickness of the frame, Fig. 5. Observations with two frames of varying thickness are sufficient to determine the actual film weight and hence the tension.

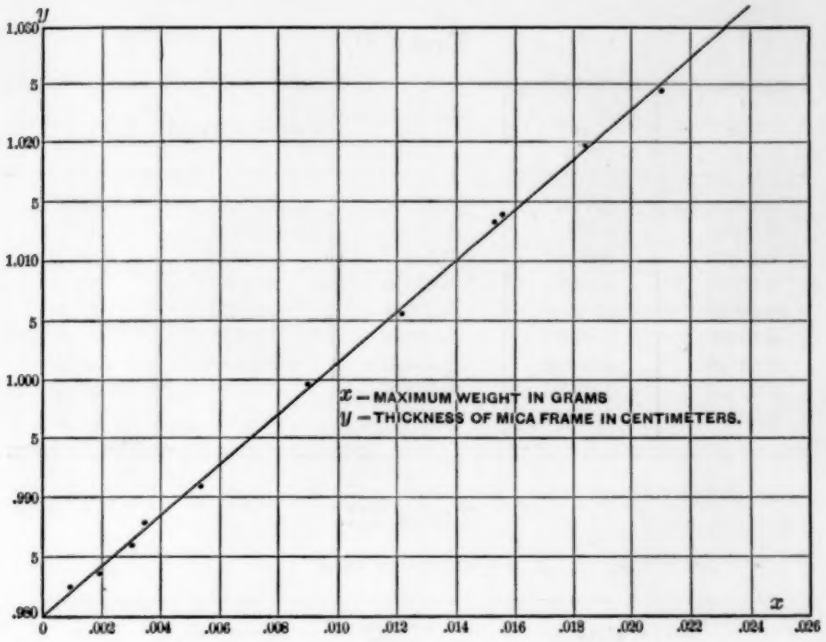

Fig. 5.

5. In the case of thin frames the tension can be determined at once from the maximum weight uncorrected, with results that vary less than do those obtained by the method of capillary tubes. For example, compare Table II. with Table III., the latter giving selected results obtained by Quincke by the capillary tube method.[1]

6. As y and t are small, a small error in the assumed value of ρ will not appreciably affect the calculated value of T, Eq. (6).

y being small, the film is much narrower than with a glass frame. Therefore there is less temperature change due to evaporation from the film surface, and less absorption of gases and impurities from the air.

7. The equations for w and y are not so complex that they cannot be used. In Table II. are given the values of T deduced by formula (8). It will be noted that the last frame is about sixteen times as thick as the first, yet the greatest difference in these values is but a little more than one part in two hundred. Of the results for the first four frames, the greatest dif-

[1] Wiedemann's Annalen, No. 5, 1894, p. 14.

ference is one part in seven hundred. The thicker frames cannot be expected to give such consistent results, as the water tends to creep in between the thin layers of which the mica sheet is made up.

TABLE II.

t	w	T by formula. $T = \frac{w}{2l}$	T by equation (8).	Temperature of water.
0.00130 cm.	0.9826 g.	0.07396	0.07365	20°.7 C.
0.00190	0.9842	0.07408	0.07374	20°.7
0.00352	0.98791	0.07437	0.07372	20°.7
0.00516	0.99094	0.07458	0.07365	20°.7
0.00928	0.99991	0.07527	0.07352	20°.8
0.01206	1.00592	0.07572	0.07355	20°.8
0.01536	1.01358	0.07630	0.07345	20°.9
0.01828	1.01973	0.07676	0.07339	21°.0
0.02067	1.02468	0.07713	0.07332	21°.0

TABLE III.

(Temperature 18°.)

Kind of glass.	Diameter of tube.	Age of tube.	T
Common Jena glass	0.5832	0 hr.	0.07528
Common Jena glass	0.5851	24 hrs.	0.07336
English flint glass	0.5390	2 mos.	0.07490
English flint glass	0.5740	0 hr.	0.07411
Fusible (soft) Jena glass	0.6440	0 hr.	0.07258
Fusible (soft) Jena glass	0.9106	12 hrs.	0.07480

In this experiment I used distilled water from the Chemical Laboratory. Subsequent tests showed that this water contained considerable organic matter.

I have almost completed an investigation of the temperature coefficient of the surface tension of water by the maximum weight and mica frame method. I hope to give the results of this work in a subsequent paper.

PHYSICAL LABORATORY OF THE INDIANA UNIVERSITY.

The Resistance of Tin-foil as Changed by Electric Waves.

By C. D. Child.

THE results of recent investigations of the effect of electric waves on the resistance of tin-foil by Haga[1] and Mizuno[2] differ from the conclusions of Aschkinass,[3] who first observed this phenomenon. He found that the resistance of a grating made of tin-foil decreased when subjected to electric waves, and was again brought back to its original resistance when jarred or heated. He attributed this to a change in the tin-foil itself. The others ascribe the variation of resistance to a change of contact between consecutive strips. In this they confirm my own experience as given at the Springfield meeting of the A. A. A. S., 1895, and briefly reviewed in the *Electrical World* for Sept. 14, 1895, p. 284. At that time I expressed the hope that the method might be made quantitative. I have failed thus far in carrying out my desires in this respect, but think that it may not be amiss to give the results of my work.

The waves with which I worked were generated from an exciter of the type used by Righi, and were about 5 cm. in length. The exciter was placed at the focus of a parabolic mirror for receiving the waves. The resistance was made by fastening a piece of tin-foil on a block of wood and slitting it in such a way that the current must follow a zigzag path back and forth across it. When the tin-foil was cut so that there was considerable distance between consecutive strips, there was no effect whatever; when it was cut with a sharp knife, so that the consecutive strips were almost in contact, the effect was large.

Several resistances were tried, made in these two ways, but no change of resistance was observed which could not be ascribed to a change of contact between consecutive strips.

To make doubly sure, I folded a narrow strip of tin-foil back and forth on itself with very thin pieces of tissue paper laid between the folds of tin-foil, so that there could be no actual contact between consecutive strips. In this way I folded the tin-foil until it made a pile 2.5 cm. high, but was unable to detect any change of resistance.

Having satisfied myself that the effect was entirely due to change of contact between adjacent strips, I next endeavored to secure more regular effect in order to make quantitative measurements by this means. There is no doubt but that such a change of resistance affords the most sensitive method of detecting electric waves. The use of iron filings by Lodge and

[1] Wiedemann's Annalen, Vol. LVI., p. 571. [2] Philosophical Magazine, Vol. 40, p. 497.
[3] Wiedemann's Annalen, Vol. LIV., p. 103.

others have shown this conclusively, but the action of the filings has been found to be quite irregular. I found that the regularity of the action of the tin-foil depended largely on the material with which it was fastened to the block of wood. For this purpose I tried several substances, such as shellac, paraffin, mucilage, and beeswax. In order to get good results, it seemed to be necessary that the substance should hold the tin-foil rather firmly, and yet not rigidly. Of those used, beeswax gave the best results. Better results were obtained by having the strips of tin-foil perpendicular to the direction of the electric oscillations.

The pieces of tin-foil used were 2.5 cm. square. I first made them of this size because I thought that there might be a resonance effect. However, difference in size did not seem to affect the results, and I continued to make them of this size for convenience. It was not possible to tell certainly whether one size was better than another, because no two pieces ever worked alike even when they were of the same size, — except that most of them were equally bad. Many of them would be constantly varying in resistance even when there was no mechanical jar nor electrical disturbance that could be detected; and even the best seemed to deteriorate with age.

For qualitative work, such as lecture experiments, the method can be made to give excellent results. Some of the pieces of tin-foil worked with considerable regularity, where the intensity of the oscillations was large, but when the intensity was small, the action was very irregular. I was attempting to work with an effect analogous to that got by a diffraction grating in light. It was not at all difficult to detect such an effect, but one could not be certain of the point at which a maximum was obtained, because of the irregularity in the working of the tin-foil. Unfortunately I could not be sure that the trouble was not largely with the exciter. The exciter was worked by an induction coil with an ordinary magnetic make and break. However, the very great difference between different pieces of tin-foil showed that the trouble was not all with the exciter. Moreover, one could scarcely expect the action to be regular. The resistance was connected so as to be one arm of a Wheatstone's bridge, and by keeping the connections closed at the same time that the tin-foil was being acted upon by the electric waves, one could easily watch the effect. In general, the effect was greater the longer the time during which the waves acted upon it, but there was by no means a constant change. A large change would often take place very suddenly, as indicated by the throw of the galvanometer needle. This was no doubt caused by some more violent oscillation of the exciter, and if it were possible to have exactly the same action of the exciter at all times, the change in the tin-foil would have been regular. One spark larger than usual would often produce a greater

effect than would be produced at another time by long-continued action of the exciter, and a method of detecting electric waves can hardly be relied upon, when some slight irregularity in the exciter is liable to produce such large effects.

However, to show of what the method is capable, two series of readings are given below, taken when endeavoring to measure wave-lengths by means of a diffraction grating. By watching the galvanometer indicating the resistance of the tin-foil, it was usually possible to bring it back to its original resistance. I would measure its resistance, allow it to be acted upon for six seconds, and again measure its resistance. While the oscillations were occurring, the connections to the tin-foil were broken as near to it as possible; for I found that otherwise waves were liable to be caught up by the wires running to the bridge and its connections, and brought to the tin-foil, thus making its action still more irregular. In one case the initial resistance was 7.3 ohms. The changes in five consecutive readings, when the circumstances were apparently the same, were approximately 0.98, 0.71, 0.73, 1.18, 0.62 ohms. At another time, when the intensity of the waves was not so great, the resistance decreased as follows, 0.28, 0.38, 0.66, 0.79, 0.38. These are measurements which I took on the effect through a grating, which we may call a diffraction grating. They are rather better than the average.

My conclusion is that the method is an excellent one for qualitative but not for quantitative work.

NEW BOOKS.

Hydrodynamics. By HORACE LAMB, F.R.S. Cambridge, University Press, 1895.

It is not difficult to understand the attraction which the study of fluid motion appears always to have had for mathematicians. Whether we look at the practical or theoretical importance of its problems, the mathematical difficulties involved in many of them, the beautiful simplicity of the solution of certain most complex motions, — as, for instance, the motion of ellipsoidal masses of fluid under their own attraction, or of two-dimensional jets with free surfaces, — and, at the same time, the large field for investigation still open, the reason for this interest is not far to seek.

The first connected treatise on the subject of any worth, in English, was Lamb's *Treatise on the Motion of Fluids*, published in 1879. This was followed, in 1888, by Basset's well-known and valuable *Treatise on Hydrodynamics*, and now we have what is practically a revised and greatly enlarged edition of Lamb's original treatise, but so altered that the author has thought it well to make a change in its title. The 258 pages of the original have expanded to 604. The chief additions are in the treatment of the Motion of Solids through Liquids (from 36 pages to 105 pages), Wave Motion (42 pages to 230 pages), Viscosity (16 pages to 84 pages), and a new chapter on the Motion of Rotating Masses of Liquid. It will thus be seen that the bulk of the new matter is on Wave Motion. So great, indeed, is this increase that this portion may be regarded as a monograph on the subject. It is, perhaps, to be regretted that the extension should be so one-sided, as the author has thereby been compelled, in order to bring the book within reasonable compass, to omit much important matter in other branches. The due selection of matter from such a wealthy storehouse of results, is not easy, and each writer will have his own preferences. In any text-book, general principles must, in any case, take a prominent place. In this respect, the treatise is a model of clearness, and cannot be too highly praised. But a mathematical text-book ought not to be a mere repertory of known facts, but an armory of weapons for further investigations. Two canons should then govern the selection of matter: is the result important in itself? does it exemplify a method? Many a problem, of small interest in itself, becomes of importance as an example of a particular method.

One could wish that, in this last respect, the treatment had been fuller. Clebsch's representation of the velocity by three functions, ϕ, m, ψ, where

$$udx + vdy + wdz = d\phi + md\psi,$$

is wanting. The general method of curvilinear co-ordinates, and the method of images, are only slightly touched on. The only example of Lagrange's method of dealing with fluid motion is the artificial wave form of Gerstner, whilst in Dirichlet and Dedekind's treatment of the motion of an ellipsoidal mass of fluid, a most striking illustration might have been given. In Chapter IV., it would have been preferable to *deduce* solutions instead of taking functions satisfying $\nabla^2\psi = 0$, and seeing what boundary conditions they satisfy. For instance, in the case of the elliptic cylinder, the opportunity is lost of introducing the method of elliptic curvilinear co-ordinates, and deducing from the general solution of $\nabla^2\psi = 0$, in these co-ordinates, the proper form for translation, rotation, or any surface motion whatever. We could have dispensed with much of the new matter on waves if thereby we could have had Kirchoff's treatment of the motion of a solid of revolution, a much fuller treatment of the motion of two bodies, of vortex motion, and especially of the motion of ellipsoidal masses of fluid. The author has apparently made it a rule to omit any investigation in which elliptic functions enter. This is unfortunate, as many solutions are most elegantly expressed in these functions — witness the motion of two parallel cylinders; it is also unnecessary, as a knowledge of the elements, at least, of these functions is possessed nowadays by all mathematicians.

One most valuable feature of the book is the number of diagrams of stream lines introduced. Nothing gives a clearer idea of what is taking place than this translation of formulæ into visible form. Compare, for instance, the stream lines for the cylinder given on pages 86 and 87 with those for the sphere on pages 137 and 265; or examine the lines of motion of a symmetrical body in a plane, in which for the first time the correct form is given. The diagram illustrating the effect of viscosity on wave motion of a finite depth, again, is most instructive. Most of these are due to the author himself. In fact, the reader already acquainted with the subject is not only pleased with the way in which old friends are introduced, but with frequently occurring bits of new work which the author modestly allows him to pick out for himself.

The two or three deviations from ordinary usage introduced by the author are scarcely to be commended. It is always a great advantage, not only in working out results, but also in interpreting them, to choose functions which have a definite physical meaning, a magnitude which is mentally seeable, and which has a direct reference to the subject we are dealing with.

Professor Lamb confesses that it is with some hesitation that he has used throughout the reversed sign for the velocity potential. His reason is that thereby it represents, when multiplied by the density, the impulsive pressure which will generate the motion from rest, and also that it is then analogous to the magnetic potential. But by the old usage it represents the flow of liquid along a line between two points, *or* the momentum per sectional area in a small uniform tube between the points. It follows at once that to generate this an impulse must be applied equal to the momentum. We can therefore much more clearly form a visual picture of the distribution of its value, and interpret results more easily with this conception, than with the new one. That it may have an analogy with the magnetic potential is no help whatever in hydrodynamical work. By all means seek for analogies; but for actual use in investigations let us employ the simplest ideas natural to the subject in hand. These remarks are illustrated again by the new definition given to a source or sink. Surely the natural measure of a source is the number of cubic centimeters or the number of grams issuing from it per second. But this would spoil the analogy with electrostatics. Therefore it is proposed that the unit source shall be one which delivers 12.566··· cubic centimeters per second. It is a curious inversion of the 4π-mania from which some eminent electricians suffer. These, instead of taking the idea natural to their subject, and defining the unit of electricity so that two units at a distance of one centimeter repel with a force of one dyne, desire to alter it so they shall repel with a force of $1/16\pi^2$ dynes, and this to make the electrostatic unit analogous to a source. For the same purpose of analogy with a foreign subject also, the letters F.G.H. are used to denote the component of the vector potential, in place of L.M.N. introduced by Helmholtz, and used by all writers in hydrodynamics since. As in the former case, mutual courtesy should induce the 4π-electricians to use L.M.N. instead of F.G.H.

On the contrary, the author conforms to common usage in one respect in which a change might be desirable. Stokes' stream function is defined as the total flux through a circle divided by 2π, so that the velocities are given by

$$u = -\frac{1}{r}\frac{d\psi}{dr}, \quad v = \frac{1}{r}\frac{d\psi}{dy}$$

There is much to be said in favor of defining it as the flux itself. ψ then has a definite meaning. It is clear at once that, since the energy due to a thin vortex (ωdS) is that due to the initiating impulse on the diaphragm filling its aperture:

The energy

= impulse × change of flux,

= momentum in closed uniform tube bent round the vortex × flux,

= circulation × flux = $\omega dS \times \psi$.

Wherever ψ enters, it tells its own tale. The only argument for the usual definition is that the expressions for the velocities are simplified. This is so, but only with the effect of complicating other expressions by bringing in 2π where it has no meaning; *e.g.* the energy is now $2\pi\omega\psi dS$.

After all, perhaps, these points are merely matters of taste. There is no doubt that in the book before us Professor Lamb has produced a most valuable addition to mathematical literature which should find a place in every mathematical library.

W. M. HICKS.

Elements of the Mathematical Theory of Electricity and Magnetism. By J. J. THOMSON, F.R.S. 8vo. pp. 510. Cambridge, The University Press, 1895.

The program, admirably carried out in this elementary treatise, is stated in the introduction. "For in the simple cases," says the distinguished author, "the absence of analytical difficulties allows attention to be more easily concentrated on the physical aspects of the question, and thus gives the student a more vivid idea and a more manageable grasp of the subject than he would be likely to attain if he merely regarded electrical phenomena through a cloud of analytical symbols."

The mathematical apparatus employed is of the simplest kind, and it is employed, not for the purpose of enabling the student to obtain results quickly by making short cuts in his physical reasoning, but only as a record of the results obtained by a purely physical method of analyzing physical phenomena. His attention is riveted upon the physical law describing what is going on in the elements of space and time, and not upon the differential equation, which is only a symbolic statement of this law, and hence "eine Nebensache," as the German saying goes.

The passage from the infinitesimal elements of the phenomena to the phenomena themselves, such as we perceive them, is not treated as a mere mathematical process of integration. The student is trained to build up synthetically the finite from the infinitely small by examining carefully what is going on in every one of the elements which he is summing up. For instance, the theorems concerning transformations of surface integrals into line integrals, or of volume integrals into surface integrals, are valuable labor-saving machines in the hands of an advanced mathematical physicist. To a young student of mathematical physics they are perplexing puzzles, unless he has been taught first how, for instance, to build up the magnetomotive force around a finite closed curve from the currents which pass through the various elementary areas of a surface bounded by that curve, etc. It is this synthetic building-up which Professor Thomson substitutes in place of

abstract mathematical theorems. Such a treatment keeps the student continually in touch with the phenomena; it makes him a mathematician by teaching him how to apply physics to mathematics.

The classification of the phenomena is the usual one; it is in accordance with the historical order of development. The statical phenomena are discussed first, the current comes next, and the phenomena of electromagnetic induction in conductors and dielectrics follow last. More than half of the book is devoted to statics. This will probably displease the engineering student; but it should be remembered that our knowledge of the two most important physical constants of the electromagnetic field, that is, of specific inductive capacity and magnetic permeability, is derived from these statical phenomena. Besides, tubes of induction, especially those of electric induction, have a much deeper meaning in Professor Thomson's view of electromagnetic phenomena, and play a much more significant part than is usually attributed to them. Our ideas concerning these tubes can be derived most easily from a study of statical phenomena. Those who have followed carefully Professor Thomson's investigations within the last few years will be delighted with his electrostatics and magnetostatics as given in this elementary treatise.

The phenomena of electrolysis, and Faraday's laws relating to these phenomena, form the foundation on which Professor Thomson builds what one may call the mechanical model of the electric current. Those who are acquainted with his theory of moving electrostatic tubes, which, as a rule, have their ends attached to the atomic charges, would naturally expect him to emphasize the intimate relation between the electric current and the chemical effects accompanying it. A very forcible and exceedingly clear statement of this relation will be found in the chapter on electric currents.

The discussion of the magnetic field which accompanies an electric current is very brief, but, nevertheless, it is clear and complete. The same may be said of the discussion of magneto-electric induction in conductors. The application of the fundamental laws of induced electromotive and of the magnetomotive force to the theory of alternating currents will probably be considered as too brief to satisfy the needs of electrical engineering students. It may be answered, however, that this is not a book on electrical engineering, and that a student who has mastered this book will find no difficulty whatever in rushing through any one of the many voluminous treatises on alternating current generators, transformers, and motors. The distribution of alternating electric and magnetic forces in the interior of conducting masses is beautifully worked out, and will be much appreciated. This is probably the first time that this extremely interesting subject has been discussed in an elementary treatise.

The chapter on electrical units is not shrouded in that sepulchral gloom which generally accompanies every discussion of units. This is due to the fact that the interesting theory of the various methods of absolute measurement has been assigned to this chapter.

The chapter containing Maxwell's electromagnetic theory in general, and the electromagnetic theory of light in particular, is so interesting that the young student will undoubtedly regret that Professor Thomson has not made this chapter longer. But he can console himself with the idea that if this book has not given him the whole story of the modern achievements of the electromagnetic theory, it has certainly prepared him to hear and appreciate this story.

An elementary book like this, by so eminent an authority as Professor Thomson, contributes almost as much to the progress of science as an important discovery. Teachers and students of electricity should not be without it.

M. I. PUPIN.

Crystallography, a Treatise on the Morphology of Crystals. By N. STORY-MASKELYNE, M.A., F.R.S. 12mo, pp. 521 + xii. Oxford, Clarendon Press, 1895.

As indicated by the sub-title, the author's aim is a complete discussion of the external form of crystals; and it may be said at the outset that nothing in English equals this book as regards detailed treatment of geometrical crystallography.

In the first chapter is given a very brief sketch of the general properties of crystals. This serves as an introduction to the more special study of their forms.

Chapters II., III., IV., and V. deal with crystal planes and axes, and with the mutual relations of the same. The treatment, as required by the nature of the case, is almost purely mathematical. It is in most respects clear and thorough, and will be found very useful to those who wish to supplement an elementary knowledge of the subject. The foundation here laid is, naturally, specially adapted to use in the later portions of the book; yet the discussion of Miller's symbols and of the possible kinds of crystal symmetry will be appreciated by any who wish to consider these subjects apart from the rest of the book.

The following chapter is divided into two sections,—one devoted to mero-symmetry (a term coined to designate all kinds of symmetry except six, viz., the highest symmetry possible in each of the six systems) and the other to twin crystals. It seems unfortunate that two subjects which are so pre-eminently matters of crystal structure should be treated from the

standpoint of their external form, but this is in accord with the plan of the book, and the author is consistent in carrying it out.

In Chapter VII., which occupies 200 pages, the six crystal systems are described in detail. All the possible crystal forms are classified and named. At the end of the description of the forms in each system, several articles are devoted to the subject of twin crystals. Though no mention is made of ogdohedrism (the author would call it ogdo-symmetry), nevertheless this peculiar type of symmetry is duly noticed under the heading "Tetarto-systematic Haplohedral Forms" in the hexagonal system.

Chapter VIII., on the measurement and calculation of the angles of crystals, contains a very good description of some kinds of goniometers and their use, as well as methods of calculation which may be of service to such as desire help in this line.

The final chapter presents an extremely satisfactory *résumé* of the various methods of representing crystals, though parts of it may be found rather too condensed to be appreciated by the beginner.

The book as a whole may be characterized as independent and thorough. Its independence lends freshness and interest, to be sure, though in several respects it may also be found by some to detract from its value as a book of reference. For instance, no allusion is made to the symbols used by Naumann, Weiss, Lévy, or Dana in designating the position of planes on a crystal, although all these are simpler than Miller's. This is no oversight. The author simply (and rightly) considers Miller's method the best, and uses it. Yet three of the above-mentioned systems of symbols are still in use by crystallographers, and the fourth is found of service to beginners.

The terms *hemihedral* and *hemihedrism* are not discussed. They are replaced by words designating the symmetry. The new nomenclature can hardly be said to be an improvement on the old, so far as the significance of its derivation is concerned, as there is no more reason for calling the symmetry of calcite a *partial* symmetry, than there would be for designating the symmetry of axinite in the same way. Each substance crystallizes with the *full* symmetry belonging to itself, without reference to the greater or less symmetry which may be manifested by other substances.

The author's use of the words *merohedral* (p. 159) and *clinopinakoid* (p. 358) in a sense entirely new is certainly not defensible, and the omission of all reference to alternative names for the six systems may also be criticized, especially in the case of the sixth, which is much more generally called triclinic than anorthic. Yet only the latter name appears in the book. Reference may also be made to the employment of the Greek letters ξ, η, and ζ instead of the more usual α, β, and γ, in designating the angles between the axes. On the whole, the book may be said to lose more than it gains by its independence in the matter of terminology. Yet

the words *haplohedral* and *diplohedral*, denoting the absence or presence of parallel planes in a crystal form, would seem to have good reason for existence.

In the matter of clearness the book is frequently at fault through a combination of long sentences with cumbersome technical words. More rarely a misprint or a word wrongly used is responsible for the difficulty. An example may be quoted from p. 156. "When we turn to the natural polyhedra presented in crystals in order to determine to what extent these actually accord in their geometrical characters with the crystalloid systems hitherto considered, we cannot fail to recognize that whereas the crystallographer, guided heretofore solely by observation and experience, referred every crystal to one or other of six crystallographic systems, those systems furnish precisely the several types of symmetry which coincide in their distinctive features with the six crystalloid types of symmetry resulting from the above principle;" viz., that there are only four possible varieties of isogonal zones in a *crystalloid system*. If the above sentence was not intended to call attention to the coincidence between the independent results of the crystallographer and of the mathematician in establishing six crystal systems, the reviewer has not yet fathomed it.

A. C. GILL.

Electricity and Magnetism, a Mathematical Treatise for Advanced Undergraduate Students. By FRANCIS E. NIPHER. 8vo, pp. 426. St. Louis, Boland Book Co., 1895.

This book occupies a field in electrical science that is of fundamental importance, but which has not been covered satisfactorily by existing literature. The older treatises, of which that of Mascart and Joubert may be taken as an example, were based upon objects and methods that antedated the remarkable development of electrical applications during recent years. It might be said that the science of electricity, considered from the purely mathematical or physical standpoint, is entirely independent of practical applications; nevertheless the fact remains that this progress, commercial though it may be, has produced the greatest stimulating effect that any science ever received. Even in such apparently abstract subjects as those of rational units and definite terminology, their appreciation and general adoption are due far more to this influence than to all others combined. Furthermore, many of the most earnest and most successful students of electrical science are those who are preparing themselves for a career in its engineering branches; therefore it is reasonable to expect that the instruction not only in technical schools, but also in courses of pure science, shall be in harmony with and lead up to the practical applica-

tions. Even those students who intend to devote themselves to the theoretical side of electricity are much benefited by and are usually anxious to acquire at least a general knowledge of the dynamo, motor, and transformer. There are many works on special subjects which are essentially mathematical in character, but so far as the reviewer is aware, this book is the first general treatment of the mathematical principles of electricity and magnetism which leads directly up to electrical engineering, but only goes into it to the extent of giving a few practical examples. Professor Nipher therefore deserves credit as a pioneer in a most important direction. His work is also to be commended for features which show ability and experience in the art of teaching. For example, the subjects of potential and energy are introduced and explained in connection with gravitation. This not only gives a definite and well-grounded conception, but also broadens the view of the electrical student, who is apt to think that potential is confined to his particular science. The frequent use of concrete numerical examples is still another great aid to the student as well as to the instructor. The scope of this book shows good judgment; that is to say, the point at which it begins and the previous knowledge assumed, as well as the limits reached, are well suited to the majority of students who are likely to use it.

Nevertheless, there are several features which are open to question or criticism. The most prominent of these is the excessive amount of matter. While four hundred pages is not an unusually large number for a text-book, nevertheless the subject is somewhat difficult and is given in condensed form, so that it would not be practicable to cover more than a few pages in one lesson. Moreover, it would have been perfectly possible to considerably reduce the length without sacrificing anything essential. For example, in the chapter on electrostatics some of the cases of potential, distribution of charge, etc., under various conditions, might have been omitted.

The old-fashioned conceptions and methods relating to magnetism, such as susceptibility, magnetic moment, etc., might have been abridged without material loss. In fact, the modern way of treating the magnetic circuit in terms of flux, M.M.F., and reluctance is not incorporated in the book as freely and completely as present custom warrants. In this connection, the idea of surface charge or distribution of magnetism used by Professor Nipher is unnatural and perpetuates the geometrical methods of treating this subject, including poles, straight lines of force, and other conditions which rarely if ever exist and differ so radically from the more modern views of the "closed loops" of Faraday.

The author's definition of current in terms of its magnetic effect is very abrupt and hardly satisfactory to the majority of minds, and the same is true of the manner in which the subject of alternating currents is introduced.

The book would have been improved by giving references to authorities and sources of information, not only to assist and encourage those who wish to pursue their studies further, but also in simple justice to writers from whom much material was derived. The fact that it is an elementary and not an advanced treatise is not a valid answer to this criticism.

In spite of these objections this book is a fairly successful attempt in a new and yet very important direction, and should be welcomed by those interested in education in electrical science, particularly when it forms part of an engineering course.

F. B. CROCKER.

Notes on the Nebular Theory. By WILLIAM FORD STANLEY. 8vo. pp. 260. London, Kegan Paul, Trench, Trübner & Co., 1895.

This book of 260 pages contains the speculations upon the Nebular Theory of a gentleman not a professional astronomer, who has evidently devoted much thought to the subject. Nevertheless, it is difficult to regard the work as a positive contribution to our knowledge of cosmogony. The author states in the preface that he has been led "to the conclusion that a modified form of the Nebular Theory of Laplace might be established on some new ideas which I had formed, and by certain calculations that I felt sure the actual conditions warranted."

The treatment of the subject is not mathematical, and, except for some fifteen pages of historical notes upon the theories of Wright, Kant, Laplace, and later philosophers of cosmogony, the book is merely a statement — and not always a clear one — of the numerous ideas that have occurred to its author. It cannot be said that these ideas are based upon our present knowledge of astronomy, mathematics, and physics, to the degree that would be expected in a book upon such a subject as this. The strong confirmation of the general theory of Laplace that has been furnished by recent spectroscopic researches seems to have been overlooked. A chapter is given to a "discussion of the mechanical principles upon which our solar-planetary system may have been formed," but many of the views advanced cannot be harmonized with the laws of mechanics. Although the margins of the reviewer's copy of the book are quite fully occupied with exceptions taken to the author's statements, yet it seems hardly worth while to cite them in detail here.

The author's style is not lucid; in fact, his language is so greatly clogged with words that it is often very difficult to grasp the idea presented, even after repeated readings of the passage. We give an example, without selecting an especially conspicuous one. "In this case, with proportional time-condensation, under the increasing amount of tangential impulse due

to centralized condensation into gravitation, which produces the law of orbit, the distances of the planets from the sun and their separate masses would be symmetrically proportional, in accordance with the pull of gravitation and the tangential momentum of the amount of the condensed matter" (p. 66). On p. 105, the term *iridescence* is used where irradiation is obviously the phenomenon referred to.

Under the chapter devoted to "Comets considered as Ordinary Gravitative Matter in Rotation constructively as a Part of the Planetary System," we read: "These ideas will be now reproduced, with some extenuations that appear to me necessary in reconsidering the subject." Among the ideas requiring "extenuation," we cite this: "If the comet depart through some disturbance from its law of original construction, its matter may present afterward only what we may term a specialized confusion, too complicated to discuss by the most advanced science" (p. 127).

Something more than the last third of the book is occupied with geological theories, upon which the reviewer can express no opinion, although decided exceptions must be taken to the astronomical speculations upon which the succession of geological periods is based, in the chapter entitled "Periodic Conditions of Earth-formation produced by effects incidental to the Nebular Clouding at Inferior Planetary Formation, and at Critical Temperatures of Matter surrounding the Sun."

The typographical appearance of the book is excellent.

EDWIN B. FROST.

Mechanics and Hydrostatics. By R. T. GLAZEBROOK, F.R.S. 8vo. pp. 244, 176, and 208. Cambridge, The University Press, 1895.

This volume consists of three parts,—Dynamics, Statics, and Hydrostatics, each with its own paging. The parts may also be obtained separately. A notice of the series and a review of Dynamics has been given in the PHYSICAL REVIEW, Vol. III., No. 3.

Analytical Chemistry. By N. MENSCHUTKIN. Translated by JAMES LOCKE. 8vo, pp. xii, 511. London, Macmillan & Co., 1895. *Received.*

Elementary Mensuration. By F. H. STEVENS. pp. 243. Macmillan & Co., 1895. (*Received.*)

Elements of Physics: a College Text-Book. By EDWARD L. NICHOLS and WILLIAM S. FRANKLIN. In three volumes. Vol. I., Mechanics and Heat. pp. 228. Macmillan & Co., 1895.

Vol. II. (Electricity) and Vol. III. (Sound and Light) are in preparation.